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D2BHM2303 (Pages: 2) Name:........................

Reg. No:....................

SECOND SEMESTER B.Sc DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH2B07T : NUMBER THEORY

Time: 3 Hours **Maximum Marks: 80**

Part A. Answer All the questions. Each carries 1 mark.

Choose the correct answer.

1. For positive integers a and b, then $gcd(a,b)\ lcm(a,b) =$ ....

   a) 1 b) ab c) $2ab$ d) 0

2. The value of k for which $3 \equiv k \ (mod\ 7)$ is ....

   a) 0 b) 4 c) 3 d) 7

3. For integers a and b, $a \equiv b \ (mod\ n)$ and if $gcd(a,n) = 2$, then $gcd(b,n) =$ ....

   a) 2 b) 4 c) 0 d)1

4. The congruence $a \equiv a \ (mod\ n)$ is true ....

   a) always b) only when $a = 0$ c) only when $a = 1$ d) only when $a = n$

5. The only prime of the form $n^2 - 4$ is ....

   a) 5 b) 7 c) 11 d)13

Fill in the Blanks. Each carries 1 mark.

6. The number of positive divisors of 12 is ....
7. For $n = 10$, $\sum_{d/10} \mu(d) =$ ....
8. The linear congruence $3x \equiv 2 \ (mod\ 5)$ has ....number of solutions modulo 5.
9. If μ denotes the Mobius function, then $\mu(6)$ is ....
10. The sum of the divisors of 6 is ....

(10 x 1 = 10 Marks)

(PTO)

Part B. Answer any 8 questions. Each carries 2 marks.

11. State Mobius inversion formula.
12. Find the binary representation of 105.
13. State and prove Euclid's lemma.
14. Define Mobius function and find $\mu(4)$.
15. If $a \equiv b \ (mod\ n)$ and if m/n then prove that $a \equiv b \ (mod\ m)$.
16. Define lcm of two nonzero integers a and b.
17. Define Pseudoprime and give an example.
18. State Fundamental theorem of Arithmetic.
19. State Fermat's theorem.
20. Find the prime factorization of the integer 1234.

(8 x 2 = 16 Marks)

Part C. Answer any 6 questions. Each carries 4 marks.

21. If $ca \equiv cb \ (mod\ n)$ then show that $a \equiv b \ (mod\ n/d)$, where $d = gcd(c, n)$.
22. Determine all positive integer solutions of the Diophantine equation $18x + 5y = 48$.
23. Show that the Mobius function μ is a multiplicative function.
24. a) If $n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$ is the prime factorization of $n > 1$, then show that $\tau(n) = (k_1 + 1)(k_2 + 1) \cdots (k_r + 1)$.

    b) Using the above formula, compute $\tau(28)$.
25. Prove that $\sqrt{2}$ is irrational.
26. State and prove Wilson's theorem.
27. Using Euclidean algorithm, find the $gcd(272, 1479)$.
28. Show that there is an infinite number of primes of the form $4n + 3$.

(6 x 4 = 24 Marks)

Part D. Answer any 2 questions. Each carries 15 marks.

29. a) Prove that there is an infinite number of primes.

    b) Are the integers 1949 and 1951 twin primes? Justify.
30. State and prove Chinese remainder theorem.
31. a) State and prove division algorithm.

    b) Show that $\frac{a(a^2+2)}{3}$ is an integer for all $a \geq 1$.

(2 x 15 = 30 Marks)