64

D2BHM2302 (PAGES 2) Reg.No............................

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH2B06T: CALCULUS II

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. The derivative of the function $lnax$ is ....

   A) $\frac{1}{ax}$ B) $\frac{1}{a}$ C) ax D) $\frac{1}{x}$

2. The series, $\sum_{n=1}^{\infty}\frac{1}{n^2}$ is ....

   A) convergent. B) divergent. C) alternating. D) none of the these.

3. The radius of convergence of the power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 = \cdots = \sum_{n=0}^{\infty} x^n$ is ....

   A) 0 B)1 C) 2 D) ∞

4. The slope of the tangent line to the curve, $x = t,\ \ y = 2t$ is.................

   A) 0 B) 1 C) 2 D) t

5. If the equation $r = f(\theta)$ is unchanged when θ is replaced by $\pi + \theta$, then the graph of $r = f(\theta)$ is symmetric with respect to.........

   A) x-axis. B) y-axis. C) polar axis. D) none of these.

Fill in the Blanks.

6. The Range of the function e^x is ....
7. The number $3.2\overline{14}$ represented as a rational number is ....
8. The series $\sum_{n=1}^{\infty} a_n$ is.................., if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$.
9. In polar coordinate system, $(r, \theta + 2\pi) = (r, \ldots\ldots\ldots)$.
10. The area of the surface of revolution in polar coordinate system is ....

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Find the value of x, for (a) $\ln x = 7t + 9$ (b)$e^{2x} = 10$.
12. State Squeeze theorem and find the value of $\lim_{n\to\infty}\frac{n!}{n^n}$.
13. Define a monotone sequence and show that the sequence $\left\{\frac{n}{e^n}\right\}_{n=1}^{\infty}$ is decreasing.
14. Find the derivative $\frac{dy}{dx}$ for the function, $e^{2x} = Sin(x + 3y)$.

(PTO)

15. Define a series and list the first five terms of the sequence $\left\{\frac{n}{n+1}\right\}$.

16. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2n-1}{3n+1}$.

17. Find the equation of the tangent line to the curve at the point corresponding to the value of the parameter, $x = 2t - 1$ and $y = t^3 - t^2$, $t = 1$.

18. Evaluate the following integral $\int \left(\frac{1+lnx}{2+xlnx}\right) dx$.

19. Find all the point of intersection of the given curves, $r = 1$ and $r = 1 + \cos\theta$.

20. Define the Horizontal and Vertical asymptote of a curve with illustrations.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Evaluate the following limits: (a) $\lim_{y\to 0^+} y\, lny$. (b) $\lim_{u\to 0^+} u\, cotu$. (c) $\lim_{z\to 0^+}\left(\frac{1}{sinz} - \frac{1}{z}\right)$.

22. Show that $\int_0^{\infty} e^{-tx}dx$, converges if $t > 0$ and diverges if $t \leq 0$, where t is a constant.

23. State Root test of convergence and hence test the convergence of the series $\sum_{n=1}^{\infty}\frac{2^{n+3}}{(n+1)^n}$.

24. Show that the alternating harmonic series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ is conditionally convergent.

25. Find the area of the region that lies outside the circle $r = 3$ and inside the cardioid, $r = 2 + 2\cos\theta$.

26. Sketch the graph of the polar equation, $r = 1 + \cos\theta$.

27. Find the area of the surface obtained by revolving the curve $x = e^t - t,\ y = 4e^{t/2},\ 0 \leq t \leq 1$ about y-axis.

28. Find the second derivative $\frac{d^2y}{dx^2}$, for the function $x = t^2 - 4, y = t^3 - 3t$.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29.(a) Show that the function $y = 2e^{-x/2} + 5e^{3x/2}$ is a solution of the differential equation,

$4y'' - 4y' - 3y = 0$.

(b) Find the absolute extrema of the function $f(x) = e^{2x} - e^x$ on $[-2,0]$.

30. State and prove integral test for convergence and hence determine whether $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ converges or diverges.

31. (a) Find the Taylor series expansion of the function $f(x) = \ln x$ at $a = 1$ and hence find its interval and radius of convergence.

(b) Find the Maclaurin series for the function $f(x) = (1 + x)^k$, where k is a real number and hence find its radius of convergence.

(2 x 15 = 30 Marks)