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D2BHM2301 (PAGES 3) **Reg.No**.....

Name: .....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH2B05T: TWO DIMENSIONAL GEOMETRY

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represent a pair of lines if.....
   a) $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
   b) $2abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
   c) $abc + 2fgh + af^2 + bg^2 + h^2 = 0$
   d) None of the above
2. The directrix of the parabola $y^2 = -4ax$ is.....
   a) $x = a$ b) $x = -a$ c) $y = a$ d) $y = -a$
3. A double ordinate through the focus of an ellipse is called.....
   a) Diameter b) Latus rectum c) Major axis d) Minor axis
4. The foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, are.....
   a) $(0, \pm ae)$ b) $(\pm ae, 0)$ c) $(0, \pm be)$ d) $(\pm be, 0)$
5. The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola if.....
   a) $h^2 = ab$ b) $h^2 < ab$ c) $h^2 > ab$ d) $h = ab$

Fill in the blanks

6. The relation connecting old coordinates and new coordinates in translation of axes is----------
7. The general equation of a parabola with focus (α, β) and directrix $ax + by + c = 0$ is -------
8. The equation of the polar of the point (x_1, y_1) with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is ----------
9. The line $y = mx + c$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if c = ----------
10. Every curve whose equation is of the second degree is a ----------

(10 x 1 = 10 Marks)

(PTO)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Define the diameter of a parabola. What is the diameter of $y^2 = 4ax$?
12. Simplify $x^2 + 2y^2 + 2x - 4y + 3 = 0$ by changing origin to $(1, -2)$.
13. Trace the parabola $x^2 = -4ay$.
14. Prove that, if the lines $ax^2 + 2hxy + by^2 = 0$ are perpendicular, then so are the lines $bx^2 + 2kxy + ay^2 = 0$.
15. Find the equation of the parabola whose focus is $(-1, -1)$ and directrix is $x + 3y + 3 = 0$.
16. Transform to parallel axes through the point (3,5) the equation
$$x^2 + y^2 - 6x - 10y - 2 = 0 .$$
17. Find the equation of bisectors of the angles between the pair of lines
$$15x^2 - xy - 6y^2 = 0$$
18. Find the centre of the conic $2x^2 - 4xy + 2y^2 + x - 3y + 12 = 0$.
19. Define auxiliary circle and eccentric angle.
20. Trace the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Find the equation of the pair of lines through the origin which represents the lines perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$.
22. Show that from any point there can be drawn two tangents, real or imaginary, to a parabola.
23. Calculate the angle between the lines $x^2 + 8xy + y^2 + 16x + 4y + 4 = 0$.
24. Find the equation of an ellipse referred to its axes as coordinate axes whose end of the minor axis is (5,0) and length of the latus rectum is $\frac{50}{13}$.
25. What does the equation $4x^2 + 2\sqrt{3xy} + 2y^2 = 1$ become when the axes are turned anticlockwise through an angle of 60^o ?
26. Find the value of λ so that the equation $\lambda x^2 + 2xy + \lambda y^2 + 4x + 4y + 3 = 0$, may represent a pair of lines.
27. Find the equation of the conic $3x^2 - 5xy + 6y^2 + 11x - 17y + 13 = 0$ when the origin is shifted to the centre.
28. If PSP' and QSQ' be any two focal chords of a conic which are at right angles to one another; prove that $\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}$ is constant.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. a) Prove that the locus of poles of normal chords of the parabola $y^2 = 4ax$ is $(x + 2a)y^2 + 4a^3 = 0$

    b) Prove that the locus of the poles of tangents to the parabola $y^2 = 4ax$ with respect to the circle $x^2 + y^2 - 2ax = 0$ is the circle $x^2 + y^2 - ax = 0$

30. a) Prove that the locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the curve $\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^2)^2$

    c) A perpendicular is drawn from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to any tangent. Prove that the locus of the foot of the perpendicular is $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$

31. Find the axis, latus rectum, vertex and focus of the parabolas.

    a) $x^2 + 2xy + y^2 - 2x - 1 = 0$

    b) $2x^2 + 4xy + 2y^2 - 2x - 3 = 0$

(2 x 15 = 30 Marks)