D2BMC2302 (PAGES 2) **Reg.No............................**

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

COMPUTER SCIENCE & MATHEMATICS (DOUBLE MAIN)

GDMA2B03T: MULTI VARIABLE AND VECTOR CALCULUS

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. Describe the domain and range of $f(x,y) = \sqrt{9 - x^2 - 4y^2}$.
2. Evaluate $f(0,0,0)$ and $f(1-t,t,-1)$ for $f(x,y,z) = x^2ye^{2x} + (x+y-z)^2$.
3. Define continuity of functions of two variable with an example.
4. Find f_x, f_y, f_z and f_{xz} for the function $f(x,y) = x^2 + 2xy^2 + yz^3$.
5. Discuss the nature of the critical point (0,0) for the surface $z = x^2 + y^2$.
6. Find the divergence of $F(x,y) = x^2yi + xy^3j$ and $G(x,y,z) = xi + y^3z^2j + xz^3k$.
7. Define Laplacian operator and harmonic function.
8. Give any two properties of double integral.
9. Evaluate $\iint_R x\cos xy\, dA$ for $R: 0 \le x \le \frac{\pi}{2}$, $0 \le y \le 1$.
10. Define line integral of a vector field.
11. Determine whether the vector field $F = (-y + e^x \sin y)\, i + [(x+2)e^x \cos y]j$ is conservative.
12. What do you mean by smooth curves?

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Evaluate $f(1,-1,1), f(-1,1,-1), \frac{d}{dx}f(x,x,x), \frac{d}{dy}f(1,y,1)$ and $\frac{d}{dz}f(1,1,z^2)$ for $f(x,y,z) = x^2ye^{2x} + (x+y-z)^2$.
14. Let $f(x,y) = \begin{cases} \frac{\cos y \sin x}{x}; x \neq 0 \\ \cos y; x = 0 \end{cases}$. Is f continuous at (0,0). Also find the value of C such that $f(x,y) = \begin{cases} \frac{3xy}{\sqrt{x^2+y^2}}; (x,y) \neq 0 \\ C; (x,y) = 0 \end{cases}$ is continuous at (0,0).
15. Use the method of Lagrange multipliers to find the maxima of the function $f(x,y) = 16 - x^2 - y^2$ subject to the constraint $x + 2y = 6$.

(PTO)

16. Find the volume of the solid bounded above by the plane $z = y$ and below in the xy plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

17. Evaluate $\int_0^1 \int_0^x (x^2 + 2y^2)\ dy\, dx$.

18. Evaluate $\iint_R x^2y^5\, dA$, where R is the rectangle $1 \leq x \leq 2,\ 0 \leq y \leq 1$, using an iterated integral with $y - integration$ first.

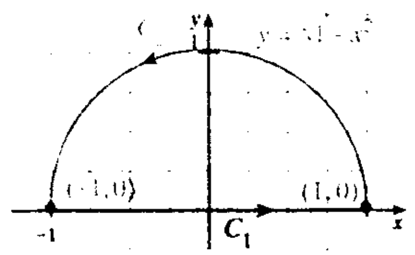
19. Evaluate $\int_C (x^2 - y^2)i + 2yzj - x^2k$, where C is the curve $x = t^2, y = 2t, z = t,\ 0 \leq t \leq 1$.

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. Find the point on the intersection of the plane $x + 2y + z = 10$ and the paraboloid $z = x^2 + y^2$ that is closest to the origin.

21.

Verify Green's theorem for $\oint_C (-ydx + xdy)$, where C is the closed path shown in the figure:



(1 x 10 = 10 Marks)