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D2BEM2302 (PAGES 2) Reg.No............................

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT2B02T: ADVANCED CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. Evaluate $\int_1^\infty e^{-x}\,dx$.
2. Give examples of (a) convergent sequence (b) divergent sequence.
3. What is absolutely convergent series? Give an example.
4. State the root test.
5. Find the radius of convergence of $\sum_{n=0}^{\infty} n!\,x^n$.
6. Obtain the Maclaurin's series expansion of $f(x) = \cos x$.
7. Describe the curve represented by $x = 4\cos\theta, y = 3\sin\theta, 0 \le \theta \le 2\pi$.
8. Show that the surface area of a sphere of radius r is $4\pi r^2$.
9. The point $(2, \frac{\pi}{3})$ is given in polar coordinates. Find its representation in rectangular coordinates.
10. Sketch the graph of the polar equation $\theta = -\frac{\pi}{4}$.
11. Find the parametric equation for the line passing through $P(1,-4,2)$ that is parallel to the vector $\vec{v} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$.
12. Find the distance between the point (3,1,2) and the plane $2x - 3y + 4z = 7$.
13. Write an equation in spherical coordinates for the paraboloid with rectangular equation $4z = x^2 + y^2$.
14. For the curve $\boldsymbol{r}(t) = t\hat{\imath} + 2t\hat{\jmath} + 3t\hat{k}, 0 \le t \le 4$ find the length of the curve.
15. Obtain unit tangent vector $\boldsymbol{T}(t)$ for the curve $t^2\hat{\imath} + t^3\hat{\jmath}$ at $t = 1$.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Show that $\lim_{n\to\infty} r^n = 0$ if $|r| < 1$.
17. State divergence test. Using divergence test show that $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$ is divergent.
18. Use integral test to determine whether $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ converges or diverges.
19. Find a power series representation for $\tan^{-1} x$ by integrating a power series representation of $f(x) = \frac{1}{1+x^2}$.
20. Find the area of the surface obtained by revolving the circle $r = \cos\theta$ about the line $\theta = \frac{\pi}{2}$.
21. Obtain an equation of the plane containing the points $P(3,-1,1), Q(1,4,2)$ and $R(0,1,4)$.
22. Sketch the graph of $\frac{y^2}{4} + \frac{z^2}{9} = 1$.
23. Find the velocity, acceleration and speed of an object with the position vector $\boldsymbol{r}(t) = e^t\hat{\imath} + e^{-t}\hat{\jmath} + t^2\hat{k}$.

(PTO)

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Check the convergence of the following series.

    (a) $\sum_{n=1}^{\infty} \frac{2n^2+n}{\sqrt{4n^7+3}}$ (b) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

25.

    (a) State and prove alternating series test.

    (b) Prove that the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent.

26.

    (a) Find the length of the cardioid $r = 1 + \cos\theta$.

    (b) Find an equation of the tangent line to the curve $x = \theta\cos\theta, y = \theta\sin\theta$ at the point where $\theta = \frac{\pi}{2}$.

27.

    (a) Find the curvature of the smooth curve described by the vector function $\boldsymbol{r}(t) = t\,\hat{\imath} + \frac{1}{2}t^2\hat{\jmath} + \frac{1}{3}t^3\hat{k}$.

    (b) Find scalar tangential and normal components of acceleration of a particle with position vector $\boldsymbol{r}(t) = 2\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath} + t\,\hat{k}$.

(2 x 10 = 20 Marks)