

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024**(Regular/Improvement/Supplementary)****MATHEMATICS****GMAT2B02T: CALCULUS AND INFINITE SERIES****Time: 2 ½ Hours****Maximum Marks: 80****SECTION A: Answer the following questions. Each carries two marks.****(Ceiling 25 Marks)**

1. Find the inverse of the function $f(x) = \frac{2x-3}{5x-7}$ where the domain of f excludes $x = \frac{7}{8}$.
2. Solve $e^{2x+3} = 10$.
3. Evaluate $\int e^{5x} dx$.
4. Differentiate $y = 3^{(x+1)}$.
5. Evaluate the expression $\log_3 \frac{1}{81}$.
6. Show that derivative of $\sinh x$ is $\cosh x$.
7. Express the number $\coth^{-1}\left(\frac{5}{4}\right)$ in terms of natural logarithms.
8. Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1}{2x}$.
9. Write the first five terms of the sequence $a_n = \frac{n-1}{n^2+1}$.
10. State Squeeze Theorem for Sequences.
11. Discuss the convergence of geometric series.
12. Find the Maclaurin series of $f(x) = \cos x$.
13. Determine the radius of convergence of $\sum_{k=0}^{\infty} \frac{k^5}{(k+1)!} x^k$.
14. Show that the series $1 + \frac{1}{2} + \frac{1}{2^2} + \dots$ converges and also find its sum.
15. What is an alternating series? Give an example.

SECTION B: Answer the following questions. Each carries five marks.**(Ceiling 35 Marks)**

16. Find the area of the region between the graphs of $y = x^2 + 2$ and $y = x - 1$ and the vertical lines $x = -1$ and $x = 2$.
17. Find the length of the graph of $f(x) = (x - 1)^{\frac{3}{2}} + 2$ on $[1, 2]$.
18. State and prove power rule.

19. Show that $\lim_{n \rightarrow \infty} (1 + h)^{\frac{1}{n}} = e$.
20. Evaluate $\int_2^4 \frac{1}{\sqrt{4-x}} dx$.
21. Prove that $\lim_{n \rightarrow \infty} |a_n| = 0$ then the sequence (a_n) converges to zero.
22. Justify the statement: Every conditionally convergent series need not be absolutely convergent.
23. Find the Taylor series for $f(x) = \ln x$ at $x = 1$ and determine its interval of convergence.

SECTION C: Answer any two questions. Each carries ten marks.

24. A solid has a circular base of radius 2. Parallel cross sections of the solid perpendicular to its base are equilateral triangles. What is the volume of the solid?
25. a) Show that $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$.
- b) The concentration of a certain drug (in mg/cc) in a patient's bloodstream t hr after injection is $C(t) = \frac{t}{5(t^2+1)}$. Determine the average concentration of the drug in the patient's bloodstream over the first 4 hr after the drug is injected.
26. State and Prove ratio test for convergence of series.
27. a) State limit comparison test and hence show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ divergent.
- b) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2n}{4n-1}$ converges or not.

(2 x 10 = 20 Marks)