

## SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

STATISTICS: COMPLEMENTARY COURSE FOR MATHEMATICS &amp; CS

GSTA2C02T: PROBABILITY THEORY

Time: 2 Hours

Maximum Marks: 60

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. What is sample space? A coin is tossed until a head appears, write down the sample space.
2. State empirical definition of probability.
3. A problem is given to two students and their chances of solving it are  $1/2$  and  $1/3$  respectively. What is the probability that the problem will be solved?
4. A can hit a target four times in 5 shots, B, three times in 4 shots, C two times in 3 shots. Calculate the probability that only one will hit the target.
5. Let  $X$  be a random variable with p.d.f  $f(x) = kx^2(1-x)$ ;  $0 < x < 1$ . Find the value of  $k$ .
6. If  $X$  has the pdf  $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$

Obtain the distribution of  $-2\log X$ .

7. If  $f(x) = \frac{1}{2^x}$ ,  $x = 1, 2, 3, \dots$

Show that  $E(2^X)$  does not exist.

8. What are the properties of moment generating function?
9. What do you mean by conditional probability function.
10. For any two random variables  $X$  and  $Y$ , show that  $E(E(X|Y)) = E(X)$ .
11. If  $X$  is a random variable with pdf  $f(x)$ , Prove that  $E(X^2) \geq [E(X)]^2$ .
12. What are the properties of distribution function?

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Twenty-five books are placed at random in a shelf. Find the probability that a particular pair of books shall be:
  - (i) Always together.
  - (ii) Never together.
14. State and prove addition theorem on probability for two events.

(PTO)

15. Find p.d.f of a random variable with distribution function  $F(x) = 1 - e^{-x}$ ,  $x > 0$ .
16. Define raw moments and central moments. Obtain the relation between raw moments and central moments.
17. Two dice are thrown.  $X$  represents the sum of the two numbers that come up. Determine  $E(X)$  and  $V(X)$ .
18. Find the mean of  $Y = X^2 + 1$  if  $X$  has probability function.

$x$ :	0	1	2	3
$P(x)$ :	0.1	0.2	0.3	0.4

19. Find the m.g.f for  $f(x) = \frac{1}{8}(1+x)$ ,  $2 < x < 4$ .

**SECTION C: Answer any one question. Each carries ten marks.**

20. (a) State and prove Bayes' theorem.
- (b) The probabilities of  $X$ ,  $Y$  and  $Z$  becoming managers are  $4/9$ ,  $2/9$  and  $1/3$  respectively. The probabilities that the Bonus Scheme will be introduced if  $X$ ,  $Y$  and  $Z$  becomes managers are  $3/10$ ,  $1/2$  and  $4/5$  respectively.
- (i) What is the probability that Bonus Scheme will be introduced?
- (ii) If the Bonus Scheme has been introduced, what is the probability that the manager appointed was  $X$ ?

21. The p.d.f of two random variables  $(X, Y)$  is given by  $f(x, y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$

Find the marginal distributions. Also find the conditional mean and variance of  $X$  given  $Y = y$ .

**(1 x 10 = 10 Marks)**