D2BMT2202 17 Reg.No.....

Name: .....

SECOND SEMESTER B.Sc DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY & CS

GMAT2C02T: MATHEMATICS -2

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 20 Marks)

1. Use the definition of sinh $x$and cosh x to show that $\cosh^2 x - \sinh^2 x = 1$
2. Find the polar equation of the circle $x^2 + (y-3)^2 = 9$
3. Find the Cartesian equation of the polar equation of $r = \frac{4}{2\cos\theta - \sin\theta}$
4. Find the directrix of the parabola $r = \frac{25}{(10+10\cos\theta)}$
5. Sketch the graph of the curve $r = 4\cos\theta$
6. If $z_1 = 4 + 3i, z_2 = 2 - 5i$ then find $z_1 z_2$ and $\frac{z_2}{z_1}$
7. Find the value of $Re\, f$ and $Im\, f$ of $f = \frac{z-2}{z+2}$
8. Check for analyticity using Cauchy Riemann equations for $f(z) = e^x(cosy + isiny)$
9. Evaluate $\int_{-i}^{i} \frac{dz}{z}$
10. Evaluate $\oint_C \frac{dz}{z^2+1}$, where C *is* $|z+i| = 1$
11. State Liouville's and Morera's theorem
12. Find the first order partial derivatives of $f(x,y) = e^{xy}\sin(xy)$

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 Marks)

13. Find all polar coordinates of the point $P\left(2, \frac{\pi}{6}\right)$.
14. Graph the curve $r^2 = 4cos\theta$.
15. Determine whether the function $u = x^2 - y^2 - y$ is harmonic or not. If harmonic find a conjugate harmonic function v of u.
16. Integrate $\frac{e^{-z}sinz}{z^2}$ counterclockwise around the unit circle.
17. Evaluate $\oint_C \frac{z^3-6}{2z-i}dz$ around the circle C: $|z| = 2$.
18. Find $\frac{dw}{dt}$at the point $t = 0$ if $w = xy + z, x = cost, y = sint, z = 6t^2$.
19. Show that the function $f(x,y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x,y) approaches (0,0).

SECTION C: Answer any one question. Each carries *ten* marks.

20. Evaluate $\int_C \frac{z^4-3z^2+6}{(z+i)^3}dz$, where C is the curve a) $|z| = 2$ b) $|z-4i| = 1$.
21. a) Find the area enclosed by the cardioid $r = 2(1+\cos\theta)$
    b) Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ if $w = x^2 + y^2 + z^2, x = e^v usinu, y = ue^v cosu, z = ue^v$

(1 × 10 = 10 Marks)