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D2BST2201 (PAGES 2) **Reg.No**.....

Name: .....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

STATISTICS: COMPLEMENTARY COURSE FOR MATHEMATICS & CS

GSTA2C02T: PROBABILITY THEORY

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. If the letters of the word " REGULATIONS" be arranged at random, what is the chance that there are exactly two letters between R and E ?
2. A purse contains 3 red balls and 4 white balls and a second purse contains 4 red and 3 white balls. If a ball is selected from one of the purses, what is the probability that it is a white ball?
3. One bag contains 5 red and 3 white balls. A second contains 4 red and 7 black balls. If one ball is drawn at random from each bag , what is the probability that both are of the same colour?
4. The p.m.f of a random variable X is given as follows:

x	0	1	2	3	4	5
P(x)	k^2	k/4	5k/4	k/4	$2k^2$	k^2

Find (i) k. (ii) Write down the distribution function of X.

5. If $f(x) = (k + 1)\, x^2$ for $0 < x < 1$ is a p.d.f, find the value of k. Obtain the distribution function of X.
6. If X has the pdf $f(x)=e^{-x}, x \geq 0$ find the p.d.fof e^{-x}.
7. Evaluate E(2^X) for the p.m.f $f(x)=\frac{1}{2^x}, x=1,2,3,...$

Write your comments about the expected value.

8. Give an example of a random variable whose m.g.f does not exist.
9. Can $M_X(t)=\frac{1}{(1-t)}$ the m.g.f of some r.v?
10. What do you mean by independence of two random variables?
11. If X and Y are two independent random variables then show that $M_{X+Y}(t) = M_X(t).M_Y(t)$.
12. For any two random variables X and Y, show that E(E(X|Y)) = E(X).

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Four cards are drawn at random from a pack of 52 cards. Find the probability that:

(i) They are king, a queen, a jack and an ace.

(ii) Two are kings and two are queens.

(iii) Two are black and two are red.

(iv) There are two cards of hearts and two cards of diamonds.

(PTO)

14. In a factory, machines A, B and C produce 2,000, 4,000 and 5,000 items in a month, out of which output 5%, 3% and 7% are defective respectively. From the factory's monthly products one is selected at random and inspected. What is the probability that it is good? If it is good, what is the probability that it is from machine C?

15. (i) Define pdf of a continuous random variable.

(ii) If the distribution function F(x) is given to be

$$F(x)=\begin{cases} \dfrac{2x^2}{5}, & if \quad 0<x\le 1 \\ -\dfrac{3}{5}+\dfrac{2\left(3x-\dfrac{x^2}{2}\right)}{5}, & if \quad 1<x\le 2 \\ 1 & if \quad x>2 \end{cases}$$

Find the density function.

16. Evaluate the pdf of Y = 2X + 4 if the pdf of X is given by

$$f(x)=\begin{cases} x; & 0<x1 \\ \dfrac{3-x}{4}; & 1\le x<3 \\ 0; & otherwise. \end{cases}$$

17. Find the median and mode of the distribution with pdf $f(x)=6(x-x^2);0<x\le 1$.

18. The probability function of X is $f(x)=\dfrac{1}{2\theta}e^{\frac{-|x-\theta|}{\theta}}, -\infty<x<\infty$. Find the m.g.f of X.

19. If $f(x,y)=3xy(x+y), 0\le x\le 1, 0\le y\le 1$,

Find E(X | Y) and V(X | Y).

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. (a) Define independence of two events.

(b) It is 9 :4 against the wife who is 45 years old living till she is 73 and 5 : 2 against her husband now 55 living till he is 80. Find the probability that:

(i) Both will be alive.
(ii) None will be alive.
(iii) Only wife will be alive.
(iv) Only husband will be alive.
(v) Only one will be alive.
(vi) At least one will be alive.

21. For the joint density function $f(x,y)=\begin{cases} \dfrac{2}{3}(1+x)e^{-y}, 0<x<1, \ y>0 \\ 0, otherwise \end{cases}$

Obtain the conditional distribution of X given Y = 1 and that of Y given X = 1/3.

(1 x 10 = 10 Marks)