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D2BHM2204 (PAGES 3) Reg.No..............................

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

HONOURS IN MATHEMATICS

GMAH2B08T: DISTRIBUTION THEORY

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. If E(X) = 5 and V(X) = 12, then $E(X^2)$ = ...

   (a) 7 (b) 17 (c) 37 (d) 60

2. If E(X) = 3, then E(4X-6) = ...

   (a) 12 (b) 6 (c) 18 (d) 13

3. X and Y are two independent random variables, f(x|y) = ...

   (a) $f_1(x)$ (b) $f_2(y)$ (c) Zero (d) f(x,y)

4. The mean and variance of an exponential distribution when $\theta > 1$ is

   (a) Mean = variance (b) Mean > variance

   (c) Mean < variance (d) Does not exists

5. The MGF of a Cauchy distribution ....

   (a) Does not Exist. (b) Exists if t < 1 (c) Exists if t > (d) Exists.

Fill in the Blanks.

6. If X is a random variable, then V(2X - 2) = ...
7. If f(x,y) is the pmf of a bivariate random variable, then $\sum_x f(x,y)$ =....................
8. X and Y are two independent random variables, Cov (X, Y) = ...
9. For a Binomial distribution with parameters (7, 0.5), its mode =................
10. If X follows Uniform over [a,b], then its variance =.................

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Evaluate E(2^X) for the following p.m.f $f(x)=\frac{1}{2^x}$, $x=1,2,3,...$

    Write your comments about the expected value.

12. Show that E(aX +b) = a E(X) + b; where a and b are constants.
13. If X and Y are two independent random variables, show that V(aX + bY) = a^2 V(X) + b^2 V(Y).
14. For any two random variables X and Y show that $(E(XY))^2 \le E(X^2)E(Y^2)$.

(PTO)

15. If the moment generating function of a Binomial distribution is $M_X(t) = \left(\frac{1}{3}\right)^5 (2+e^t)^5$, obtain the mean and variance of a distribution.

16. State the inter relation among Exponential and Gamma distribution.

17. If $Z \rightarrow N(0,1)$ find $P(-2 < Z < 1.5)$.

18. If X follows normal with mean 30 and SD 5. find.

(i) $P[26 \le X \le 40]$ (ii) $P[X \ge 45]$.

19. State the Weak law of large numbers.

20. Define convergence in distribution.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. If X is ar.v having the p.d.f $f(x) = \frac{x+1}{2}, -1 \le x < 1$, find E(X) and V(X)

22. Define moment generating function. Given f(x) = C x , 0 < x < 2. Determine C and $M_X(t)$.

23. The joint p.d.f of (X , Y) is given in the table.

(i) Find the marginal distributions.

(ii) Examine whether X and Y are independent.

X \ Y	0	1	2
-1	$\frac{1}{15}$	$\frac{3}{15}$	$\frac{2}{15}$
0	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
1	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{2}{15}$

24. Let X be a discrete random variable having the pdf $f(x) = 2^{-x}; x = 1, 2, 3, \ldots$ Obtain its mgf. Hence find mean of the distribution.

25. If *X* and *Y* are independent Poisson random variables, find the conditional distribution of X given X + Y.

26. Obtain the m.g.f. of an exponential distribution and hence find its mean and variance.

27. Examine whether WLLN holds for the sequence X_n of independent random variables.

$$P(X_n = \frac{1}{\sqrt{n}}) = \frac{2}{3} \quad , \quad P(X_n = \frac{-1}{\sqrt{n}}) = \frac{1}{3}$$

28. Let X_i assumes the values i^α and $-i^\alpha$ with equal probability. Check whether the law of large numbers hold to the independent variables $X_1, X_2, \ldots$ for $\alpha < \frac{1}{2}$.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. If $f(x,y)=\begin{cases} k(x+y+xy), 0<x<1,\ 0<y<1 \\ 0, otherwise \end{cases}$

Determine

(i) The constant k (ii) The marginal densities of X and Y

(iii) The conditional density of X and Y.

30. (i) Prove that $\mu_{r+1} = \lambda\left[r\,\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]$, where μ_{r+1}, μ_{r-1}, μ_r are the central moments of Poisson distribution with parameter λ . Hence obtain the first four central moments.

(ii) Derive MGF of Poisson distribution.

31. (i) Define Normal distribution. What are the important properties of Normal distribution?

(ii) If X is normally distributed with mean 11 and SD 1.5. Find the number k such that

(a) P(X> k) = 0.3 and (b) P(X < k) = 0.09

(2 x 15 = 30 Marks)