61

D2BHM2203 (PAGES 2) **Reg.No..........................**

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

HONOURS IN MATHEMATICS

GMAH2B07T: NUMBER THEORY

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. Which one of the following is true for size estimate for p_n(nth prime number).

   a) $p_n \leq p_1 \cdots p_{n-1} - 1$ b) $p_n \geq p_1 \cdots p_{n-1} - 1$

   c) $p_n = p_1 \cdots p_{n-1} - 1$ d) $p_n \neq p_1 \cdots p_{n-1} - 1$

2. In how many ways can the even integer 78 be represented as the sum of odd primes.

   a) 6 b) 7 c) 8 d) 9

3. An example of prime triplets $(p, p+2, p+6)$ is.................

   a) 41, 43, 46 b) 10, 29, 49 c) 81, 83, 87 d) 47, 49 , 59

4. Identify the incongruence equation.

   a) $38 \equiv 6(mod\ 4)$ b) $4 \equiv 2(mod\ 4)$ c) $40 \equiv 1(mod\ 13)$ d) $6 \equiv 6(mod\ 7)$

5. Identify the congruence modulo n equation.

   a) $40 \equiv 2(mod\ 13)$ b) $4 \equiv 2(mod\ 4)$ c) $41 \equiv 1(mod\ 13)$ d) $59 \equiv 10(mod\ 7)$

Fill in the Blanks.

6. The $gcd(128,125,120)$ is ....
7. Find x for the equation $59 \equiv x(mod\ 7)$.
8. Solve for x, $25 \equiv x(mod\ 16)$
9. The sum of the divisors of 450 is ....
10. The τ and σ functions of 12 is ....

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Justify the statement: The cube of any integer has one of the forms: $9k,\ 9k+1,\ 9K+8$.
12. Assuming that $gcd(a,b) = 1$, Prove that $gcd(a-b, a+b) = 1\ or\ 2$.
13. Define e-prime numbers with an example.
14. State Bonse's inequality.
15. Prove that if $a \equiv b(mod\ n)$ and $b \equiv c(mod\ n)$then $a \equiv c(mod\ n)$.

(PTO)

16. State the divisibility test of 11 and hence check whether 149235678 is divisible by 11.

17. State and prove the converse of Wilson's theorem.

18. Define μ function with an example.

19. If f is a nonzero multiplicative function the $f(1) = 1$.

20. Find the number and sum of divisors of 180.

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Prove that the expression $\frac{a(a^2+2)}{3}$ is an integer for all $a \geq 1$.

22. Find $lcm(3054, 12378)$.

23. Using Sieve of Eratosthenes find all primes between 40 and 240.

24. Show that for a positive integer N, $9/N$ if and only if $9/S$ where $N = a_m 10^m + a_{m-1} 10^{m-1} + \cdots .. a_1 10 + a_0; 0 \leq a_k < 10$and $S = a_0 + a_1 + \cdots a_m$.

25. Show that $n^7 - n$ is divisible by 42.

26. Verify whether 10585 is an Absolute Pseudoprime.

27. Prove that the product of the positive divisors of an integer $n > 1$ is equal to $n^{\frac{\tau(n)}{2}}$

28. For $n = 434$ verify that $\sigma(n+2) = \sigma(n) + 2$. Also, what can you conclude from n and $n + 2$?

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. Using Euclidean algorithm find the $gcd(858, 325)$ and also represent the gcd as a linear combination of 858 and 325.

30. Prove the following:

(a) The product of two or more integers of the form 4n+1 is of the same form.

(b) There are infinite number of primes of the form 4n+3.

31. Prove that the linear congruence $ax \equiv b(mod\ n)$ has a solution if and only if d/b where $d = gcd(a, n)$. If d/b then it has d mutually incongruent solutions modulo n. Hence prove that if $gcd(a, n) = 1$ then the linear congruence has a unique solution modulo n.

(2 x 15 = 30 Marks)