

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

HONOURS IN MATHEMATICS

GMAH2B06T: CALCULUS II

Time: 3 Hours

Maximum Marks : 80

PART A: Answer all the questions. Each carries one mark.

Choose the correct answer.

1. Range of e^x is.....
 (a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) Not defined
2. The Harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n}$ is.....
 (a) Convergent (b) Divergent (c) Neither converges nor diverges (d) Inconclusive
3. For the sequence $\{\frac{1}{n}\}$, which of the following is not true?
 (a) Bounded above (b) Bounded below (c) Decreasing (d) Increasing
4. If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1, a_n \neq 0 \forall n$, then the series $\sum_{n=1}^{\infty} a_n$ is
 (a) Convergent (b) Divergent (c) Inconclusive (d) Neither convergent nor divergent
5. The radius of convergence of the power series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$ is
 (a) 1 (b) 0 (c) ∞ (d) 2

Fill in the Blanks.

6. The derivative of $f(x) = \log_3 x$ is
7. If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ is.....
8. If a power series $\sum_{n=0}^{\infty} a_n(x - c)^n$ converges only at $x = c$, then radius of convergence of this power series is.....
9. The graph of $r = f(\theta)$ is symmetric with respect to the polar axis, if $f(\theta) = \dots\dots\dots$
10. True or false;
 "If $P(r_1, \theta_1)$ and $P(r_2, \theta_2)$ represent the same point in polar coordinate, then $\theta_1 = \theta_2$."
 (10 x 1 = 10 Marks)

PART B: Answer any eight questions. Each carries two marks.

11. Express $\ln(0.056)$, interms of $\ln 5$ and $\ln 7$.
12. Evaluate $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$
13. Define a series and list the terms of the sequence (a) $\left\{ \frac{n}{n+1} \right\}$ (b) $\left\{ \frac{\sqrt{n}}{2^{n-1}} \right\}$
14. Express the number $-0.\overline{23}$ as a rational number.
15. Find $\lim_{n \rightarrow \infty} \left(e^{\sin \frac{1}{n}} \right)$
16. Find a power series representation for $\frac{1}{(1-x)^2}$ on $(-1,1)$, by differentiating a power series of $(x) = \frac{1}{1-x}$.

(PTO)

17. Show that alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is not absolutely convergent.

18. Describe the curves represented by the parametric equation,

$$x = a \cos \theta, \quad \text{and } y = a \sin \theta, \quad a > 0, \quad \text{with parameter interval}$$

$$(a) [0, \pi] \quad (b) [0, 2\pi]$$

19. Find the equation of the tangent line to the curve $x = \sec t, y = \tan t, -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

20. Find the rectangular coordinate representation for the following polar coordinate,

$$(a) \left(4, \frac{\pi}{6}\right) \quad (b) \left(2, \frac{\pi}{3}\right)$$

(8 x 2 = 16 Marks)

PART C: Answer any six questions. Each carries four marks.

21. Prove that (a) $\lim_{x \rightarrow \infty} \ln x = \infty$.

$$(b) \lim_{x \rightarrow 0^+} \ln x = -\infty.$$

22. Find the derivative of (a) $\text{Sinh}^{-1}x$ (b) $\text{Cosh}^{-1}x$.

23. Explain and plot the graph of $\tanh x$.

24. Find the absolute extrema of the function $f(x) = e^{2x} - e^x$ on $[-2, 0]$.

25. Evaluate (a) $\int_1^{\infty} \frac{1}{x^2} dx$ (b) $\int_0^{\infty} \cos x dx$

26. Let $f(x) = e^x$, find the Maclaurin series of f and determine its radius of convergence and interval of convergence.

27. Find the equation of the tangent line to the curve at the point corresponding to the value of the parameter,

$$(a) x = 2t - 1 \quad \text{and } y = t^3 - t^2, \quad t = 1.$$

$$(b) x = \theta \cos \theta \quad \text{and } y = \theta \sin \theta, \quad \theta = \frac{\pi}{2}.$$

28. Convert the polar equation into rectangular equation,

$$(a) r \sin \theta = -3 \quad (b) 2r \cos \theta + 3r \sin \theta = 6$$

(6 x 4 = 24 Marks)

PART D: Answer any two questions. Each carries fifteen marks.

29. (a) Determine the following series converges or diverges,

$$(i) \sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}} \quad (ii) \sum_{n=1}^{\infty} \frac{\sin n}{\sqrt{n^3+1}}$$

(b) Find the value of 'p' for which $\int_1^{\infty} \frac{1}{x^p} dx$ is convergent.

30. (a) Define the convergence of a power series, radius of convergence of the power series and the interval of convergence of the power series.

(b) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{\sqrt{n+1}}.$$

31. (a) Define the area of the surface of revolution in polar coordinate system.

(b) Find the area S of the surface obtained by revolving the circle $r = \cos \theta$ about the line $\theta = \frac{\pi}{2}$.

(c) Find the area of the region that lies outside the curve $r = 1 - \sin \theta$ and inside the curve $r = 1$.

(2 x 15 = 30 Marks)