

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

HONOURS IN MATHEMATICS

GMAH2B05T : TWO DIMENSIONAL GEOMETRY

Time :3 Hours

Maximum Marks: 80

PART A: Answer all the questions. Each carries one mark.

Choose the correct answer.

- The two lines given by $ax^2 + 2hxy + by^2 = 0$ are perpendicular if and only if
 a) $a - b = 0$ b) $a + b = 0$ c) $a - b = 1$ d) $a + b = 1$
- The length of the latus rectum of the parabola $y^2 = 4ax$ is
 a) a b) $2a$ c) $4a$ d) None of the above
- The foci of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, are.....
 a) $(0, \pm ae)$ b) $(\pm ae, 0)$ c) $(0, \pm be)$ d) $(\pm be, 0)$
- The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if.....
 a) $h^2 = ab$ b) $h^2 < ab$ c) $h^2 > ab$ d) $h = ab$
- Every curve whose equation is of the second degree is.....
 a) A circle b) A pair of lines c) A conic d) All the above

Fill in the Blanks.

- The relation connecting old coordinates and new coordinates in rotation of axes is -----
- The condition when the line $y = mx + c$ becomes a tangent to the parabola $y^2 = 4ax$ is ----
- The equation of the chord of contact of tangents drawn from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
- The equation of a directrix to the conic $\frac{1}{r} = 1 + e \cos \theta$ is.....
- The equation of a conic in polar form is

(10 x 1 = 10 Marks)

PART B: Answer any eight questions. Each carries two marks.

- Transform to parallel axes through the point (2,3) the equation

$$x^2 + y^2 - 4x - 10y - 2 = 0$$

- Simplify $2x^2 + 3y^2 + 2x - 2y + 1 = 0$ by changing origin to $(-1, -2)$.
- Find the value of λ for which the two lines $5x^2 - 14xy + \lambda y^2 = 0$ are perpendicular to one another.
- Trace the parabola $x^2 = 4ay$.
- Define the latus rectum and focal length of a parabola.

(PTO)

16. Find the locus of the middle points of chords of the parabola $y^2 = 4ax$ which pass through the point $(a, 0)$.
17. Define subtangent and subnormal of a point P to the parabola $y^2 = 4ax$.
18. Trace the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $b > a$
19. Find the centre of the conic $4x^2 + 5xy + 3y^2 + 12x - 10y + 22 = 0$.
20. Prove that the semi-latus rectum of any conic is a harmonic mean between the segments of any focal chord

(8 x 2 = 16 Marks)

PART C: Answer any six questions. Each carries four marks.

21. Find the condition that one of the two lines $ax^2 + 2hxy + by^2 = 0$ may be perpendicular to one of the lines given by $a'x^2 + 2h'xy + b'y^2 = 0$
22. Find the equation of the parabola whose vertex is $(-1, -2)$, axis vertical; passes through $(3, 6)$
23. Prove that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2$.
24. Tangents are drawn at the ends of a normal chord of the parabola $y^2 = 4ax$. Prove that the locus of their point of intersection is $(x + 2a)y^2 + 4a^3 = 0$.
25. Two lines are at right angles to one another, and one of them touches the parabola $y^2 = 4a(x + a)$ and the other to $y^2 = 4a'(x + a')$. Prove that the locus of the point of intersection of the lines is $x + a + a' = 0$.
26. Show that the sum of the focal distances of any point P on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, from foci S and S', is constant and is equal to $2a$.
27. Prove that the line $x \cos \alpha + y \sin \alpha = p$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $a^2 \cos^2 \alpha + b^2 \sin^2 \alpha = p^2$
28. Find the equation of the conic $2x^2 - 4xy + 2y^2 + x - 3y + 12 = 0$ when the origin is shifted to the centre.

(6 x 4 = 24 Marks)

SECTION D: Answer any two questions. Each carries fifteen marks.

29. Derive the formula to find the bisectors of angles between the lines $ax^2 + 2hxy + by^2 = 0$, also find the bisectors of angles between the lines $2x^2 + 4xy + 3y^2 = 0$.
30. a) Find the eccentricities, foci, directrices and latera recta of the ellipse $25x^2 + 4y^2 = 100$
 b) Find the locus of the poles of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with respect to the parabola $y^2 = 4ax$.
31. Trace the conic $2x^2 + 3xy - 2y^2 - 7x + y - 2 = 0$. Calculate the eccentricity of the conic.

(2 x 15 = 30 Marks)