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**D2BEM2202** (PAGES 2) **Reg.No.....**

**Name: .....**

**SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023**

**(Regular/Improvement/Supplementary)**

**ECONOMICS & MATHEMATICS (DOUBLE MAIN)**

**GDMT2B02T: ADVANCED CALCULUS**

**Time: 2 ½ Hours** **Maximum Marks: 80**

**SECTION A: Answer the following questions. Each carries *two* marks.**

**(Ceiling 25 Marks)**

1. Define increasing sequence. Give an example of an increasing sequence which is convergent.
2. Evaluate $\lim_{n\to\infty} \frac{\ln n}{n}$
3. Does the series $\sum \frac{n}{n+1}$ converge? Justify your answer.
4. Evaluate $\int_1^{\infty} \frac{1}{x} dx$
5. Define absolutely convergent series. Give an example of a series which is convergent but not absolutely convergent.
6. Find the radius of convergence of the series $\sum \frac{x^n}{n+1}$
7. Find the Maclaurin series representation of $f(x) = \sin x$.
8. Sketch the curve with parametric equations $x = 2\sin\theta,\ y = 2\cos\theta, 0 \le \theta \le 2\pi$.
9. Find the slope of the tangent line to the curve $x = t^2 + 1,\ y = t^2 - t$ at $t = 1$.
10. Convert the polar equation $r^2 = 4r\cos\theta$ to a rectangular equation.
11. Find the angle between the two planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$
12. Convert the cylindrical coordinates $\left(3, \frac{\pi}{4}, 3\right)$ into rectangular coordinates.
13. Find the domain of the vector function $\boldsymbol{r}(t) = < \frac{1}{t}, \sqrt{t} >$.
14. Find the unit tangent vector for the curve C defined by $\boldsymbol{r}(t) = \boldsymbol{i} + t\boldsymbol{j} + t^2\boldsymbol{k}$.
15. Define bounded sequence. Give an example of a sequence which is bounded but not convergent.

**(PTO)**

**SECTION B: Answer the following questions. Each carries *five* marks.**

**(Ceiling 35 Marks)**

16. Determine whether the integral $\int_0^\infty e^{-x} \sin x \, dx$ converges or diverges. If it converges, find its value.
17. Prove that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
18. Show that the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$ is convergent.
19. Find the area of the region enclosed by the cardioid $r = 1 + \cos\theta$
20. Find an equation in spherical coordinates for the paraboloid with rectangular equation $4z = x^2 + y^2$.
21. Find a vector function describing the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $x + y + 2z = 1$.
22. Find the curvature of $\boldsymbol{r}(t) = 2t\boldsymbol{i} + 2t\boldsymbol{j} + \boldsymbol{k}$.
23. Sketch the graph of surface: $\frac{x^2}{4} + \frac{z^2}{9} = 1$.

**SECTION C: Answer any *two* questions. Each carries *ten* marks.**

24. Discuss the convergence of the $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for all real values of $p$. Is the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ convergent? Justify your answer.
25. Find the Taylor series representation of $f(x) = \frac{1}{1+x}$ at $x = 2$.
26. Find the area of the surface obtained by revolving the curve $x = t^3$, $y = t^2$, $0 \leq t \leq 1$ about the x-axis.
27. A shell is fired from a gun located on a hill 100m above a level terrain. The muzzle speed of the gun is 500m/sec, and its angle of elevation is 30°.
    a) Find the range of the shell.
    b) What is the maximum height attained by the shell?
    c) What is the speed of the shell at impact?

**(2 × 10 = 20 Marks)**