13(A)

D2BST1801 (S4) (PAGES 2) **Reg. No............................**

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023
(Supplementary – 2018 Admission)

STATISTICS: COMPLEMENTARY COURSE FOR MATHEMATICS & CS
ASTA2C02T: PROBABILITY DISTRIBUTIONS

Time: 3 Hours **Maximum Marks:** 80

PART A: Answer all the questions. Each carries 1 mark.

1. Define V(aX), where a be a constant and random variable X.
2. First central moment of X is…
3. If the fourth central moment of a random variable Y is 10, what can you say about the kurtosis of Y?
4. If two random variables X and Y are independent, then their joint PDF is………..
5. If the correlation coefficient between two random variables is zero, can we conclude that they are independent?
6. What is the expectation of a bivariate random variable?
7. Provide an example of a discrete distribution that demonstrates the lack of memory property.
8. How can the Cauchy distribution be derived from the standard normal distribution?
9. Calculate the mean, variance, and moment generating function (MGF) of a Binomial distribution with parameters n = 10 and p = 0.3.
10. In the Central Limit Theorem (Lindberg-Lévy version) for independent and identically distributed variables, as the sample size increases, the distribution of the standardized sample mean approaches a _______ distribution.
11. The weak law of large numbers guarantees that the sample mean approaches the population mean as the sample size goes to _______.
12. According to Chebyshev's inequality, for any random variable, the probability that it deviates from the mean by more than _______ standard deviations is at most _______.

(12 x 1=12 marks)

PART B: Answer all the questions. Each carries 2 marks.

13. Define mathematical expectation in univariate analysis.
14. Define the characteristic function and explain its utility.
15. How is conditional probability defined for a bivariate random variable?
16. Define a bivariate random variable.
17. Describe the process of deriving the Lognormal distribution from the normal distribution.
18. What is the probability mass function (PMF) of the Bernoulli distribution?
19. Provide an overview of the properties associated with the normal distribution.
20. Derive mean and variance for continuous uniform distribution over zero and one.
21. Let $X \sim B(n,p)$ then derive the distribution of $n - X$.

(9 x 2 =18 Marks)

(PTO)

PART C: Answer any 5 questions. Each carries 6 marks.

22. Consider a continuous random variable Y with PDF f(y) = 2y, where y ∈ [0, 1]. Find the first four raw moments.
23. Consider a bivariate random variable (X, Y) with the joint PDF given by f(x, y) = 4xy, where x ∈ [0, 1] and y ∈ [0, 2]. Determine whether X and Y are independent.
24. Derive MGF of Normal distribution.
25. State and prove lack of memory for continue case.
26. Derive row moments of Lognormal distirbution.
27. Derive binomial distribution from Poisson distribution.
28. Let's consider a random variable X with a mean of 50 and a standard deviation of 10. Use Chebyshev's inequality to find the minimum proportion of values within 3 standard deviations from the mean.

(5 x 6 = 30 Marks)

PART D: Answer any 2 question. Each carries 10 marks.

29. Given the moment-generating function M(t) = 1 + 2e^t, find the skewness of the random variable.
30. Consider a dataset consisting of the heights (in inches) and weights (in pounds) of a group of individuals. The dataset is as follows: Height (X): [65, 68, 70, 63, 67, 72, 69, 66, 64, 71] Weight (Y): [150, 155, 160, 145, 158, 165, 156, 148, 143, 162] Calculate the correlation coefficient between height and weight for this group of individuals.
31. Derive beta first kind distribution from two independent gamma distributions.
32. State and prove Chebyshevs inequality

(2 x 10 = 20 Marks)