17(A)

Reg. No:.....

D2BMT1802(S4) (PAGES 2) Name:.....

SECOND SEMESTER B. Sc DEGREE EXAMINATION, APRIL 2023

(Supplementary-2018 Admission)

MATHEMATICS: Complementary Course for Physics, Chemistry and C.S.

AMAT2C02T - MATHEMATICS

Time: 3 Hours **Maximum Marks : 80**

PART A: Answer all questions. Each carries ONE mark.

1. $\sinh 2x =$ .....
2. What is the derivative of $x^2 \cosh x$.
3. Is $\sinh x$ periodic function. Justify your answer.
4. Find $\int \sinh \frac{x}{3} dx$.
5. Find the real part and imaginary part of $\sin z$.
6. Define analytic function.
7. Define interior point in a complex plane.
8. Find $\lim_{z \to 2+i} z^3 + 2z + 1$.
9. State Cauchy-Riemann Equations.
10. What is the domain of $f(x, y) = \frac{2x}{x^2 - y^2}$.
11. Give an example of an unbounded subset of $\mathbb{R}^2$.
12. Find the partial derivative of $x^2 + 2xy$ with respect to y.

(12 x 1 = 12 Marks)

PART B: Answer any NINE questions. Each carries TWO marks.

13. Find the derivative of $6 \sinh \frac{x}{3}$.
14. Find $\int_0^{2\sqrt{3}} \frac{dx}{\sqrt{4 + x^2}}$.
15. Show that $f(z) = |z|^2$ is differentiable only at $z = 0$.
16. Find $\int_C Re\, z\, dz$ where C vertically from $1 + i$ to $1 + 2i$ then horizontally to $3 + 2i$.
17. Evaluate $\int 6 \cosh(\frac{x}{2} - ln3) dx$.
18. Find the point of intersection of $r^2 = 4 \cos\theta$ and $r = 1 - \cos\theta$.

19. State Liouville's Theorem.

20. Show that $|z - 4i| + |z + 4i| = 10$ represents an ellipse whose foci are $(0, \pm 4)$.

21. Find the $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ at $(2, -3)$ if $f(x, y) = x^2 + 4x^2y + xy^2$.

22. State Cauchy's integral formula.

23. At what points (x, y) in the plane are the function $f(x, y) = \dfrac{1 + x}{x^2 - 3x + 2}$ is continuous?

24. Find $\lim\limits_{(x,y,z)\to(1,4,3)} x^2 + y^2 + z^2 + 1$.

(9 x 2 = 18 Marks)

PART C: Answer any SIX questions. Each carries FIVE marks.

25. Find the length of the cardioid $1 - \cos\theta$.

26. Find a polar equation of the circle $x^2 + (y - 2)^2 = 4$.

27. Graph the cardioid $r = 1 - \cos\theta$.

28. Find the diretrix of the parabola $r = \dfrac{25}{10 + 10\cos\theta}$.

29. Evaluate $\displaystyle\int_C \frac{z + 1}{z^2 - 1}$ where C is the Contour $|z| = 2$.

30. Prove that $|\mathbf{z_1} + \mathbf{z_2}| \leq |\mathbf{z_1}| + |\mathbf{z_2}|$.

31. Evaluate $\displaystyle\int_C \frac{\sin z}{(2z - 1)^3}$, Where C is the unit circle.

32. Integrate $f(z) = \dfrac{2z}{(z^2 - 1)}$ around the circle $|z| = 3/2$.

33. Show that $f(x, y) = \dfrac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches $(0, 0)$.

(6 x 5 = 30 Marks)

PART D: Answer any TWO questions. Each carries TEN marks.

34. (a). Evaluate $\displaystyle\int_0^{1/3} \frac{6dx}{\sqrt{1 + 9x^2}}$.

(b). Find the area of the surface generated by revolving $r^2 = \cos 2\theta$ about y axis.

35. (a). Find the harmonic conjugate of the function $u = x^2$.

(b). Find $\lim\limits_{z\to 5} z^2 + 1$.

36. (a). Evaluate derivative of $u = xy$ with respect to t, where $x = acost$ and $y = bsint$.

(b). Find the total differential of $f = x^2y + 2xy$ at $(1, 0)$.

(2 x 10 = 20 Marks)