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D2BMC2202 (PAGES 2) Reg.No............................

Name: ....

SECOND SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

COMPUTER SCIENCE & MATHEMATICS (DOUBLE MAIN)

GDMA2B03T: MULTI VARIABLE AND VECTOR CALCULUS

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 20 Marks)

1. Determine $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y,z) = sin^{-1}(xyz)$.
2. Find the total differential of the function $f(x,y,z) = 2x^3 + 5y^4 - 6z$.
3. Define differentiability of two variable function at a point with an example.
4. Find the gradient of a function $f(x,y) = x^2y + y^3$.
5. State Lagrange's theorem.
6. If $w = xy$ and $x = \cos t \quad y = \sin t$, find $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$.
7. Define vector field in $\mathbb{R}^3$ with example.
8. Find the critical point of the function $f(x,y) = x^3 + y^3 - 12x - 3y + 5$.
9. Compute $\iint_R (2-y)dA$, where R is the rectangle with vertices (0, 0), (3, 0), (3, 2), (0, 2).
10. Find Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ where $u = x + 2y$ and $v = x - y$.
11. Evaluate the line integral $\int_C (x^2 + y^2)ds$ for C: $x = e^{-t}\cos t, \quad y = e^{-t}\sin t, \quad 0 \le t \le \frac{\pi}{2}$.
12. Show that the vector field $f(x,y) = 2xy\, \boldsymbol{i} + x^2\, \boldsymbol{j}$ is conservative.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 Marks)

13. Let f be a function defined by $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$.

    Is f(x, y) continuous at (0,0). Explain.

14. Show that $\frac{\partial^3 f}{\partial x\, \partial y\, \partial z} = \frac{\partial^3 f}{\partial z\, \partial y\, \partial x}$ for the function $f(x,y,z) = xyz + x^2y^3z^4$.
15. Find the extreme of the function $f(x,y) = 1 - x^2 - y^2$ subject to the constraint $x + y = 1$ with $x \ge 0, \quad y \ge 0$ exist.
16. Find the divergence and curl of the vector field $x\,\mathbf{i} + y^3z^2\,\mathbf{j} + xz^3\,\mathbf{k}$.
17. Change the order of integration $\int_{-1}^{2}\int_{x^2-2}^{x} f(x,y)dy\,dx$.
18. State (i) Gauss- Divergence theorem. (ii) Stoke's theorem. Explain all terms.
19. Using Green's theorem to evaluate $\int_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(PTO)

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. (a) Find the directional derivative of the function $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $3\mathbf{i} - 4\mathbf{j}$.

(b) Find the equation for the tangent plane and the normal line to the cone $z^2 = x^2 + y^2$ at the point where $x = 3,\ y = 4$ and $z > 0$.

OR

21. (a) Evaluate $\int_0^4 \int_0^{\sqrt{4y-y^2}} \frac{1}{\sqrt{x^2+y^2}}\, dx\, dy$ by converting to polar co-ordinates.

(b) Find the volume of the tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the co-ordinate planes $x = 0, y = 0$ and $z = 0$.

(1 × 10 = 10 Marks)