D1BMT2201 (S2) (PAGES 2) **Reg. No..............................**

Name: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Improvement/Supplementary)

MATHEMATICS

GMAT1B01T: BASIC LOGIC & CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. State Rolle's theorem.
2. Define horizontal asymptote of the graph of a function.
3. Find the point of inflection of $f(x) = (x-1)^{\frac{1}{3}}$.
4. Define antiderivative of a function.
5. What do you mean by indirect proof?
6. Define quantifiers. Differentiate between universal quantifiers and existential quantifiers.
7. Define the definite integral of a continuous function defined on an interval [a,b].
8. Evaluate the sum

$$\sum_{k=1}^{10} 3.$$

9. Evaluate each Boolean expression, where $a = 3$, $b = 4$ and $c = 6$.

   (i) $[\sim (a > b)] \wedge (b < c)$

   (ii) $\sim[(a \leq b) \vee (b > c)$

10. Let $f(x) = \sqrt{4 - x^2}$. Find $\lim_{x \to -2^+} f(x)$ and $\lim_{x \to -2^-} f(x)$.
11. Find the values of x for which the function $f(x) = x^8 - 3x^4 + x + 4 + \frac{x+1}{(x+1)(x-2)}$ is continuous.
12. Let $y = f(x)$ where f is a differentiable function. Define differentials dx and dy.
13. Find the extrema of a function, if any, by examining its graph.

    $f(x) = x^2, -1 \leq x \leq 2$

14. Show that if F is an antiderivative of f on an interval I , then every antiderivative of f on I has the form G(x) = F(x) + C, where C is a constant.
15. Find

$$\int (2x + 3\sin x)\, dx.$$

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Find the extreme values of the function $f(x) = 3x^4 - 4x^3 - 8$ on [-1,2]
17. Determine the intervals where the function $f(x) = x + 1/x$ is increasing and where it is decreasing.
18. Find the slant asymptote of the graph of $f(x) = \frac{2x^2-3}{x-2}$.
19. Prove directly that the product of any two odd integers is an odd integer.
20. Define converse, inverse and contrapositive. Give converse, inverse and contrapositive of the given implication.
    $p \to q: If\ \Delta ABC\ is\ equilateral, then\ it\ is\ isosceles.$
21. Prove the sum law for limits: If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then
    $$\lim_{x\to a}[f(x) + g(x)] = L + M$$
22. Find the intervals where the following functions are continuous:
    (a) $f(x) = \cos(\sqrt{3}\,x + 4)$.
    (b) $g(x) = x^2 \sin\frac{1}{x}$.
23. Evaluate:
    $$\lim_{n\to\infty}\sum_{k=1}^{n} 3\left[\left(\frac{k}{n}\right)^2 + 2\right]\left(\frac{4}{n}\right).$$

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Using the laws of logic simplify the Boolean expression $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$
25. Find:
    (i) $\lim_{x\to 0}\frac{\sin 2x}{3x}$
    (ii) $\lim_{x\to 0}\frac{\tan x}{x}$
    (iii) $\lim_{x\to 0}\frac{\cos x - 1}{x}$
26. Prove that $\lim_{x\to 0}\frac{1}{x^2} = \infty$.
27. State and prove the fundamental theorem of calculus, part 1.

(2 x 10 = 20 Marks)