D1BHM2302 (S1) **Reg. No.............................**

Name: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH1B02T: CALCULUS I

Time: 3 Hours **Maximum Marks: 80**

Part A. Answer *all* the questions. Each question carries *one* mark.

Choose the correct answer.

1. $\lim_{x\to 0} \frac{tan\, x}{x} = \ldots\ldots\ldots\ldots\ldots$

   a) 1 b) 0 c) π d) π/2

2. Which of the following is inflection point of the function $f(x) = x^3 - 1$?

   a) (0,0) b) (0,-1) c) (1,0) d) (-1,0)

3. Let $f(x) = x^2 - 2x$. Which of the following is true?

   a) *f* is increasing in (-α,1) and decreasing in (1,α)

   b) *f* is decreasing in (-α,1) and increasing in (1,α)

   c) *f* is increasing in (-α,α)

   d) *f* is decreasing in (-α,α)

4. Which of the following is not an asymptote of the function $f(x) = \frac{1}{x^2-3x+2}$?

   a) y = 0 b) x = 1 c) x = -1 d) x = 2

5. $\lim_{x\to\infty}(2x^3 - x^2 + 1)$ = ____________.

   a) 0 b) α c) 2 d) 1

Fill in the blanks.

6. $\int x^{1/4}\, dx$ = _________.
7. $\sum_{k=1}^{10} 2k + 1 =$ ______________.
8. The area bounded by the lines $y = x, x = 2$ and the *x*-axis is ____________.
9. Let f be a continuous non-negative function on $[a, b]$, and let R be the region under the graph of on the interval $[a, b]$. The volume of the solid of revolution generated by revolving R about the x-axis is _____________.
10. Let f be smooth on $[a, b]$. Then the arc length of the graph of f from $P(a, f(a)$to $Q(b, f(b))$ is _____________.

(10 × 1 = 10 Marks)

(PTO)

Part B. Answer any *eight* questions. Each carries *two* marks.

11. Find $\lim_{x\to -3}\frac{x^2+2x-3}{x^2+4x+3}$.

12. Find the differential of the function $f(x) = xtanx$ at $x = \frac{\pi}{4}$.

13. Find the absolute maximum and absolute minimum values, if any, for the function $f(x) = x^3 + 3x^2 + 1$ on [-3, 2].

14. Verify Mean Value Theorem for the function $f(t) = \frac{sint}{1+cost}$, on $\left[0,\ \frac{\pi}{2}\right]$.

15. Show that the rectangle of maximum area that can be inscribed in a circle of fixed radius a is a square.

16. Evaluate $\int \frac{t^2-2\sqrt{t}+1}{t^2}\, dt$.

17. Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n^3}(2k+1)^2$ after first finding the sum (as a function of n) using the summation formulas.

18. Evaluate $\int_{-2}^{1}(x^3+2x)dx$.

19. Find the area of the region bounded by the graphs of $y = x^3$ and $y = x$.

20. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on [0,2] about the -axis.

(8 × 2 =16 Marks)

Part C. Answer any *six* questions. Each carries *four* marks.

21. Show that the function $f(x) = |x|$ is differentiable everywhere except at $x = 0$.

22. Determine the intervals where the function $f(x) = x + \frac{1}{x}$ is increasing and where it is decreasing.

23. Find the relative extrema of $f(x) = x^3 - 3\,x^2 - 24x + 32$ using the Second Derivative Test.

24. Determine where the graph of the function $f(x) = x - \sqrt{1-x^2}$ is concave upward and where it is concave downward. Also, find all inflection points of the function.

25. State and prove the Mean Value Theorem for Integrals.

26. Evaluate $\int_{-\pi/4}^{\pi/4}(cosx+1)tan^3x\, dx$.

27. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

28. Find the arc length of the graph of the given equation $y = -2x + 3$ from P(-1,5) to Q (2,-1) .

(6 × 4 = 24 Marks)

Part D. Answer any *two* questions. Each carries *fifteen* marks.

29. Sketch the graph of the function $f(x) = \begin{Bmatrix} -2x+4 & if\ x<1 \\ 4 & if\ x=1 \\ x^2+1 & if\ x>1 \end{Bmatrix}$. Also evaluate

a. a) $\lim_{x\to 1^-} f(x)$ and b) $\lim_{x\to 1^+} f(x)$

30. Sketch the graph of the function $f(x) = \frac{1}{1+sinx}$ with the help of differentials.

31. Evaluate the integral $\int_{-1}^{3}(4-x^2)\ dx$ as the limit of sum.

(2 × 15 = 30 Marks)