D1BHM2301 (S1) **Reg. No.....**

Name: .....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH1B01T: THEORY OF EQUATIONS AND COMPLEX NUMBERS

Time: 3 Hours **Maximum Marks: 80**

Part A: Answer all the questions. Each carries *one* mark.
Choose the correct answer.

1. If $f(x) = 3x^3 - 2x^2 + 3x + 4$, then $f(-2)$ is _____.
   a) -32. c) -34.
   b) 32. d) None of the above.
2. One possible integral root of $x^5 - 8x^4 - 5x^3 + 15x^2 + 3x - 20 = 0$ is _____.
   a) 3 c) 4
   b) 6 d) None of the above
3. The cubic resolvent in Ferrari's method is:
   a) $y^3 - by^2 + (ac - 4d)y + 4bd - a^2d - c^2 = 0$
   b) $2y^3 - by^2 + (ac - 4d)y + 4bd - a^2d - c^2 = 0$
   c) $y^3 + by^2 + (ac - 4d)y + 4bd - a^2d - c^2 = 0$
   d) None of the above.
4. If $z = -6 - 8i$, then $|z|$ is:
   a) 100 c) 10
   b) -10 d) -14
5. The polar form of $z = -2i$ is:
   a) $2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$
   b) $2\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$
   c) $2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$
   d) None of the above

Fill in the blanks.

6. Any number satisfying the proposed equation is called ----------
7. If a is a root of multiplicity α, then, in the factorization of $f(x)$ the factor $x - a$ occurs ---------- times
8. The method to solve biquadratic equation is ----------
9. The number of variations in the sequence
   $-2, -3,\ 4,\ 4, -1,\ 7,\ 7,\ 8, -5, -6, -7$ is----------
10. $(4 + 2i)(-2 - i) =$ ----------

(10 x 1 = 10 Marks)

(PTO)

Part B: Answer any *eight* questions. Each carries *two* marks.

11. Define a connected set, give an example.
12. Write a cubic equation with the roots $1, 1+i, 1-i$.
13. Give the cubic resolvent in Ferrari's method to solve a biquadratic equation.
14. Define *interior point* of a set.
15. Find an upper limit of the positive roots of the equation:
    $4x^5 - 15x^4 - 8x^3 + 6x^2 + 12x - 10 = 0$.
16. Verify that the equation $3x^3 - 2x - 7 = 0$ has a root in the interval $(1,2)$.
17. State Descartes' rule of signs.
18. Find $(3x^2 - 4x + 1)(5x + 1)$.
19. Find a lower limit of the negative roots of the equation:
    $4x^5 - 8x^4 - 5x^3 + 6x^2 + 10x - 25 = 0$.
20. Verify that $(3+i)(3-i)\left(\frac{1}{5} + \frac{i}{10}\right) = 2 + i$.

(8 x 2 = 16 Marks)

Part C: Answer any *six* questions. Each carries *four* marks.

21. Give the exponential form of $\frac{i\sqrt{2}}{4+4i}$.
22. Show that $Arg\ (z_1 z_2) \neq Arg\ z_1 + Arg\ z_2$.
23. Solve $z^3 = 8i$.
24. Using Cardan's formula, solve $x^3 + 9x - 6 = 0$.
25. Verify that the equation $x^4 - 6x^3 + 5x^2 + 14x - 4 = 0$ have roots in the intervals $(-2,-1), (0,\ 1), \left(3, \frac{7}{2}\right), \left(\frac{7}{2}, 4\right)$.
26. Show that for all real values of λ the equation
    $$(x-1)(x-3)(x-5)(x-7) + \lambda(x-2)(x-4)(x-6) = 0$$
    has all roots real and simple and separate them.
27. Find $\frac{2+5i}{-2-3i} + \frac{2i}{3i-1}$.
28. Using Taylor's formula expand $f(x) = 4x^5 - 6x^4 + 3x^3 + x^2 - x - 1$ in powers of $x + 2$.

(6 x 4 = 24 Marks)

Part D: Answer any *two* questions. Each carries *fifteen* marks.

29. a) Show that $f(x) = -x^6 - 3x^5 + 3x^4 + 11x^3 - 6x^2 - 12x + 8$ is divisible by $x^2 + x - 2$.

    b) By synthetic division find the quotient and the remainder when dividing $4x^6 + 6x^5 - 5x^4 + x^3 - 5x + 10$ by $x + 2$.

    c) Using Horner's process expand $f(x) = 4x^5 + 5x^4 - 3x^3 + 3x^2 - 2$ in powers of $x - 1$

30. Solve the biquadratic equation $x^4 - 4x^2 + x + 2 = 0$,

    transforming the equation to the form:

$$\left(x^2 + \frac{a}{2}x + \frac{y}{2}\right)^2 = \left(\frac{a^2}{4} - b + y\right)x^2 + \left(-c + \frac{ay}{2}\right)x + \left(-d + \frac{y^2}{4}\right)$$

    Using the resolvent equation:

$$y^3 - by^2 + (ac - 4d)y + 4bd - a^2d - c^2 = 0$$

31. How many real roots do the following equations have?

    a) $f(x) = x^6 + 3x^5 + x^3 - 2x^2 + x - 2 = 0$

    b) $(x) = 3x^5 + 2x^3 - x^2 + x - 1 = 0$

    c) $(x) = 1 - 4x + 3x^2 - 6x^3 + 5x^4 = 0$

(2 x 15 = 30 Marks)