D1BEM2102 (S3) **Reg. No.....**

Name: .....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Improvement/Supplementary)
ECONOMICS & MATHEMATICS (DOUBLE MAIN)
GDMT1B01T: BASIC CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. Suppose that $y = 2x^3 - x + 1$. Find Δx and Δy when:
   (a) x changes from 3 to 3.01.
   (b) x changes from 3 to 2.9.
2. State Rolle's theorem.
3. Write an integral giving the area of the surface obtained by revolving the graph of $y = \sin x$ on the interval $[0,\frac{pi}{2}]$ about the x-axis.
4. Write an integral giving the arc length of the graph of the function $y = x^2$ from *P(−1, 1)* to *Q(2, 4)*.
5. If $\cosh x = \frac{5}{4}$, find the values of the other hyperbolic functions at x.
6. Find $\lim_{x\to\infty} \frac{1}{x-1}$, $\lim_{x\to-\infty} \frac{1}{x-1}$, and the horizontal asymptote of the graph of $f(x) = \frac{1}{x-1}$.
7. Show that if *F* is an antiderivative of *f* on an interval *I*, then every antiderivative of *f* on *I* has the form *G(x) = F(x) + C*, where *C* is a constant.
8. Find $\int 2x^3 dx$.
9. Write the sum $\sum_{k=1}^{15}(-1)^k k^3$ in the expanded form.
10. Define the area under the graph of a continuous nonnegative function defined on an interval [a, b] as the limit of a sum.
11. Use the laws of logarithms to expand the expression $\ln\left[\sqrt{x}|\cos x|(x+1)^{-\frac{1}{3}}\right]$.
12. Write an integral giving the arc length of the graph of the function $y = \frac{1}{x^2+1}$ from *P(−1, 1/2)* to *Q(2, 1/5)*.
13. Given that $\ln 2 \approx 0.6931$, and $\ln 5 \approx 1.6094$, use the laws of logarithms to approximate the expression $\ln\frac{20}{\sqrt{3}}$.
14. Let $(x) = \frac{1}{x^2}$. Evaluate the limit, if it exists:
    $a)\ \lim_{x\to 0^-} f(x)$ $b)\ \lim_{x\to 0^+} f(x)$ $c)\ \lim_{x\to 0} f(x)$
15. Find the extrema of the function, if any, by examining its graph where:
    $f(x) = x^2$ [-1≤ x ≤ 2].

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Evaluate a) $\int(x+1)3^{x^2+2x}\,dx$. b) $\int 3^t + t^3 \;\; dt$.
17. Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.
18. Show that the function $f(x) = x^3 + x + 1$ has exactly one zero in the interval [-2,0].
19. Find $\int(x=1)(x^2-2)dx$.
20. Let $f(x) = \begin{cases} -x+3 & if\ x<2 \\ \sqrt{x-2}+1 & if\ x\geq 2 \end{cases}$. Find $\lim_{x\to 2} f(x)$ if it exists.
21. Find the area of the region between the graphs of $y = x$and $y = x^3$.
22. Find the area of the surface obtained by revolving the graph of $x = \frac{1}{6}y^3 + \frac{1}{2y}$ for [1,2] about the y-axis.
23. Let $f(x) = \frac{2x^2-x+1}{3x^2+2x-1}$. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$, and find all horizontal asymptote of the graph of f.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Prove that $\lim_{x\to 2} x^2 = 4$.
25. Sketch the graph of the function $f(x) = \frac{x^2}{x^2-1}$.
26. A car moves along a straight road with velocity function $v(t) = \; t^2 + t - 6, 0 \leq t \leq 10$ where $v(t)$is measured in feet per second.
    (a) Find the displacement of the car between t = 1 and t = 4.
    (b) Find the distance covered by the car during this period of time.
27. Differentiate the functions:
    I. a) $f(x) = (\cosh x - \sinh x)^{2/3}$ b) $y = e^{\sinh 2t}$.
    II. a) $g(x) = \tanh^{-1}(\cosh x)$ b) $f(x) = \sqrt{2 + \coth 3x}$.

(2 x 10 = 20 Marks)