D1BMT2301 (PAGES 2) **Reg.No.....**

Name: .....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT1B01T: BASIC LOGIC & CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. Let $p: \Delta ABC\ is\ equilateral$ and $q: \Delta ABC\ is\ isosceles$. Write the statements $p \to q$ and $q \to p$.
2. Negate the proposition, where x is an arbitrary integer $(\forall x)(x^2 > 0)$.
3. Prove that there is a positive integer that can be expressed in two different ways as the sum of two cubes.
4. Find $\lim_{x\to 2} f(x)$ if it exists. Where $f(x) = \begin{cases} 4x+2 & if\ x\neq 2 \\ 4 & if\ x=2 \end{cases}$.
5. State Squeeze Theorem.
6. Give precise definition for limit of a function at a number.
7. Find the extrema of a function, if any, by examining its graph. $f(x) = x^2, -1 \le x < 2$.
8. State first Derivative Test.
9. Determine the intervals where the graph of $f(x) = x^4 - 4x^3 + 12$ is concave upward and the intervals where it is concave downward.
10. Find $\lim_{x\to 1^-} \frac{1}{x-1}$ and $\lim_{x\to 1^+} \frac{1}{x-1}$ and the vertical asymptote of the graph of $f(x) = \frac{1}{x-1}$.
11. Find all antiderivatives of $f(x) = 1$ on $(-\infty, \infty)$.
12. Evaluate $\int \frac{1}{x^3}\, dx$.
13. Write the sum $\sum_{k=1}^{20} \frac{1}{(k+1)^2}$ in the expanded form.
14. Evaluate the sum $\sum_{k=1}^{10} 3k^2(2k+1)$.
15. Give an example to show that even though the number of subintervalsapproach infinity, the norm of a partition may not approach 0.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Determine whether or not each a tautology $(i)\ p \vee (\sim p) (ii)\ p \wedge (\sim p)$.
17. Prove that there is a prime number > 3.
18. Let $f(x) = \begin{cases} -x+3 & if\ x<2 \\ \sqrt{x-2}+1 & if\ x\geq 2 \end{cases}$. Find $\lim_{x\to 2} f(x)$ if it exists.
19. Find (i) $\lim_{x\to \frac{\pi}{2}} x \sin x$ (ii) $\lim_{x\to \frac{\pi}{4}} (2x^2 + \cot x)$.

20. Let $f(x) = 2\sin x + \sin 2x$. Write f^1(x) and $\int f(x)dx$.
21. Let $f(x) = x^3$
    (a) Show that f satisfies the hypotheses of the Mean Value Theorem on [-1,1].
    (b) Find the number(s) c in (-1,1) that guaranteed by the Mean Value Theorem.
22. Find the vertical asymptotes of the graph of $f(x) = \frac{x}{x^2-x-2}$.
23. Evaluate $\int_{-1}^{3}(4 - x^2)\,dx$ by finding the limit of Riemann sum.

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SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Determine the truth value of each proposition, where P(x,y): $y < x^2$.
    (i) $(\forall x)(\forall y)P(x,y)$
    (ii) $(\exists x)(\exists y)P(x,y)$
    (iii) $(\forall x)(\exists y)P(x,y)$
    (iv) $(\forall y)(\exists x)P(x,y)$
    (v) $(\exists x)(\forall y)P(x,y)$
    (vi) $(\exists y)(\forall x)P(x,y)$
25. Prove that $\lim_{x\to 2} x^2 = 4$.
26. A man has 100 ft of fencing to enclose a rectangular garden in his backyard. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.
27. State and prove the mean value theorem for integrals.

(2 x 10 = 20 Marks)