D1BHM2302 (PAGES 3) **Reg.No.............................**

Name: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH1B02T: CALCULUS I

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

1. $\lim_{x\to a} \cos x =$ _______.
2. $\lim_{x\to a}[f(x) + g(x)] =$ __________.

   a. $\lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ b. $\lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

   c. $\lim_{x\to a}[f(x) \pm g(x)]$ d. $\lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. Point of discontinuity of the function $f(x) = \frac{1}{(x-2)}$ is/are _________.
4. The General Power Rule states that $\frac{d}{dx}[f(x)]^n =$ __________.
5. State whether the following statements is TRUE or FALSE.

   All critical points are extrema.
6. Verify the following function satisfies hypothesis of Rolle's Theorem

   $$f(x) = x^3 - 9x \qquad [-3,3]$$
7. $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n =$
8. $\int \frac{1}{x^3} dx =$ ______.
9. $\int (x + 1)(x^2 + 1)\, dx =$___________.
10. Area of the region bounded by the graphs of y=x, y=0 and x=2 is _________.

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. Find $\lim_{x\to 2} f(x)$ if it exists, where *f* is the piece wise-defined function.

    $$f(x) \begin{cases} 4x + 8 & if\ x \neq 2 \\ 4 & if\ x = 2 \end{cases}$$
12. State the Squeeze Theorem.
13. Show that |x| is continuous everywhere.

(PTO)

14. The edge of a cube was measured and found to be 3 in. with a maximum possible error of 0.02 in. Find the approximate maximum percentage error that would be incurred in computing the volume of the cube using this measurement.

15. Suppose f is differentiable on an open interval I. Then.

a. the graph of f is concave upward on if f' is ______ (increasing / decreasing) on I.

b. the graph of f is concave downward on if f' is ________ (increasing / decreasing) on I.

16. Let c be a critical number of a continuous function f in the interval (a, b) and suppose that f is differentiable at every number in (a, b) with the possible exception of c itself

a. If $f'(x) > 0$on (a, c) and $f'(x) < 0$ on (c, b), then has a _____ at c.

b. If $f'(x) < 0$on (a, c) and $f'(x) < 0$ on (c, b), then has a _____ at c.

17. Evaluate.

a. $\int_2^7 3\,dx$

b. $\int_0^1 (x^2 - 4)\,dx$

18. Given that $\int_{-1}^3 f(x)\,dx = 5$ and $\int_{-1}^3 g(x)\,dx = -2$, evaluate the following.

a. $\int_{-1}^3 \quad [f(x) + g(x)]$

b. $\int_{-1}^3 \quad [f(x) - g(x)]$

19. Find the area of the region bounded by the graphs of $x = y^2$and $y = x - 2$.

20. Use differentials to obtain an approximation of the arc length of the graph of

$y = \frac{1}{3}x^2 + \frac{1}{4x}$from P(1, 3) to Q(1.1, 3.52)

(8 x 2 = 16 Marks)

PART C: Answer any *six* questions. Each carries *four* marks.

21. Let *f(x)*= $\begin{cases} ax + b & if\ x < 1 \\ 4 & if\ x = 1 \\ 2ax - b & if\ x > 1 \end{cases}$

Find the values of a and b that will make *f* continuous on (-∞, ∞)

22. Let $f(x) = 2x^3 + x$

a. Find $f'(x)$ using first principle.

b. What is the slope of the tangent line to the graph of at (2, 18)?

c. How fast is changing when x = 2.

23. Prove the following using Mean Value Theorem.

If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

24. Find the critical numbers of $f(x) = x - 3x^{1/3}$.

25. Find the relative extrema of the function using second derivative test.

$$f(x) = x^3 - 3x^2 - 24x + 32$$

26. Show that $F_1(x) = x^3$, $F_2(x) = x^3 + 1$ and $F_3(x) = x^3 - \pi$ are anti-derivative of $f(x) = 3x^2$.

27. Find f if,

$$f''(t) = 2\,\mathrm{sint} + 3\cos t, \qquad f\left(\frac{\pi}{2}\right) = 1, \qquad f'\left(\frac{\pi}{2}\right) = 2$$

28. The velocity function of a car moving along a straight road is given by $v(t) = t - 20$ for $0 \le t \le 40$, where $v(t)$is measured in feet per second and t in seconds. Show that at *t = 40* the car will be in the same position as it was initially.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29. Define the $\varepsilon - \delta$ definition of infinite limit. Use this to prove $\lim_{x\to 0} \frac{1}{x^2} = \infty$

30. Prove that;

a. If f is odd then $\int_{-a}^{a} f(x)\,dx = 0$

b. If f is even then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

31. The axes of two right cylinders, each of radius r, intersect at right angles. Find the volume of the resulting solid that is common to both cylinders.

(2 x 15 = 30 Marks)