D1BHM2301 (PAGES 2) **Reg. No.....**

Name: .....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH1B01T: THEORY OF EQUATIONS AND COMPLEX NUMBERS

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer *all* the questions. Each carries *one* mark.

1. Define integral rational function with an example.
2. Without actual division show that $x^4 + 3x^3 + 3x + 2$ is divisible by $x + 2$.
3. Find limits of roots for $x^4 - 7x^3 + 10x^2 - 30 = 0$.
4. Find upper limit of the moduli of roots for $2x^4 - 7x^3 + 6x^2 - 5 = 0$.
5. State location of roots theorem for real polynomials.
6. Verify that the equation $x^3 - 7x + 7 = 0$ has roots in intervals$(-4, -3), \left(1, \frac{3}{2}\right), \left(\frac{3}{2}, 2\right)$.
7. Separate roots of $3x^4 - 4x^3 - 6x^2 + 12x - 1 = 0$.
8. State Descartes' rule of signs.
9. Show that $Re(iz) = -Imz$ and $Im(iz) = Rez$.
10. Reduce $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ to real number.

(10 x 1 = 10 Marks)

PART B: Answer any *eight* questions. Each carries *two* marks.

11. By synthetic division, find the quotient and reminder in the division of $(n-1)x^n - nx^{n-1} + 1$by$(x-1)^2$.
12. Determine k and solve $2x^3 - 6x^2 + 3x + k = 0$ if one root is twice the sum of the others.
13. Solve the equation $x^3 - 3x^2 - 9x + 27 = 0$ (Equation has multiple roots).
14. Solve $x^4 - 8x^2 - 4x + 3 = 0$.
15. Solve trigonometrically $x^3 - 3x^2 + 1 = 0$.
16. Find the rational roots of the equation $6x^3 - x^2 + x - 2 = 0$.
17. Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$.
18. Sketch set of points determined by $|z - 4i| \geq 4$.
19. Find principal argument of $z = \frac{i}{-2-2i}$.
20. Show that $|Re(2 + \bar{z} + z^3)| \leq 4$ when $|z| \leq 1$.

(8 x 2 = 16 Marks)

(PTO)

PART C: Answer any six questions. Each carries *four* marks.

21. Derive Taylor's formula.
22. Examine whether the equation $x^3 - 106x - 420 = 0$ has integral roots or not.
23. Find the number of real roots of the equation $1 - 2x + 3x^2 - \cdots + (2n+1)x^{2n} = 0$.
24. Show that an odd degree polynomial with real coefficients has at least one real root.
25. Prove that $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$.
26. Find the square roots and cube roots of $1 - \sqrt{3}$ i.
27. Show that any point z_0 of a domain is an accumulation point of that domain.
28. Show that if c is any n^{th} root of unity other than unity, then $1 + c + c^2 + \cdots + c^{n-1} = 0$.

(6 x 4 = 24 Marks)

PART D: Answer any *two* questions. Each carries *fifteen* marks.

29.
   a) By method of detached coefficients divide $x^5 - 3x^2 + 6x - 1$ by $x^2 + x + 1$.
   b) Find sum of squares of roots of $2x^4 - 6x^3 + 5x^2 - 7x + 1 = 0$.
   c) Solve the equation $8x^3 - 36x^2 + 22x + 21 = 0$,roots are in arithmetic progression.

30. Solve $x^3 + x^2 - 2 = 0$ using Cardan's formula.

31.
   a) Show that the necessary and sufficient condition for an equation $x^3 + px + q = 0$ to have three real and distinct roots is $4p^3 + 27q^2 < 0$.
   b) For what value of A has the equation $(x+3)^3 - A(x-1)^2 = 0$ three real roots?

(2 x 15 = 30 Marks)