

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023**(Regular/Improvement/Supplementary)****COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)****GDMA1B01T: CALCULUS****Time: 2 Hours****Maximum Marks: 60****SECTION A: Answer the following questions. Each carries *two* marks.****(Ceiling 20 Marks)**

1. Find $\lim_{x \rightarrow \frac{\pi}{2}} x \sin x$.
2. Let $f(x) = \sqrt{4 - x^2}$, then find $\lim_{x \rightarrow 2^+} f(x)$.
3. Find the extrema of the function $f(x) = x^2$.
4. Find $\int x^2 dx$.
5. Prove that $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$.
6. Let $f(x) = \frac{2x^2 - x + 1}{3x^2 + 2x - 1}$, find $\lim_{x \rightarrow \infty} f(x)$.
7. Find the derivative of the function $F(x) = \int_{-1}^x \frac{1}{1+t^2} dt$.
8. Suppose that $\int_1^6 f(x) dx = 8$ and $\int_4^6 f(x) dx = 5$. What is $\int_1^4 f(x) dx$?
9. Let $\int_0^1 x^2 dx = \frac{1}{3}$, then evaluate $\int_0^1 (x^2 - 4) dx$.
10. Find $\lim_{x \rightarrow \infty} \frac{1}{x-1}$ and $\lim_{x \rightarrow -\infty} \frac{1}{x-1}$.
11. Find the area of the region between the graphs of $y = e^x$ and $y = x$ with $x = 0$ and $x = 1$.
12. Find the volume of a right pyramid with a square base of side b and height h .

SECTION B: Answer the following questions. Each carries *five* marks.**(Ceiling 30 Marks)**

13. Sketch the graph of the function f defined by

$$f(x) = \begin{cases} 3 - x & , \quad x < 1 \\ 1 & , \quad x = 1 \\ 2 + \sqrt{x - 1} & , \quad x > 1 \end{cases}$$

Use your graph to find $\lim_{x \rightarrow 1^+} f(x)$, $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1} f(x)$.

14. Find $\lim_{x \rightarrow 2} (2x^2 - 4x^2 + 3)$ using limits of polynomial functions.
15. Find all the vertical asymptotes of the graph of $f(x) = \tan x$.

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16. Find the points of inflection of $f(x) = x^4 - 4x^3 + 12$.
17. Write $\sum_{k=1}^n \left(1 + \frac{k}{n}\right)^3 \left(\frac{1}{n}\right)$ in expanded form.
18. Compute the Riemann sum for $f(x) = 4 - x^2$ on $[-1,3]$ using five sub-intervals ($n=5$) and choosing the evaluation points to be the midpoints of the sub-intervals.
19. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. a) Find the volume of the solid obtained by revolving the region bounded by $y = \sqrt{x}$ and $y = x$ about the x-axis.
- b) Show that $\int_a^b x dx = \frac{1}{2}(b^2 - a^2)$.
21. a) A man has 100ft of fencing to enclose a rectangular garden in his backyard. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.
- b) At a distance of 12,000ft from the launch site, a spectator is observing a rocket being launched vertically. What is the speed of the rocket at the instant when the distance of the rocket from the spectator is 13000ft and is increasing at the rate of 480ft/sec?

(1 x 10 = 10 Marks)