D1BEM2301

Reg.No...........................

Name: ....

FIRST SEMESTER B.Sc DEGREE EXAMINATION,NOVEMBER 2023
ECONOMICS & MATHEMATICS
GDMT1B01T-BASIC CALCULUS

Time : 2.5 Hours **Maximum: 80 Marks**

SECTION A: Answer the following questions. Each carries 2 marks
(Ceiling 25)

1. Let $f(x) = \begin{cases} 2x - 4 & \text{if } x < 4 \\ x - 2 & \text{if } x \geq 4 \end{cases}$; Evaluate $\lim_{x \to 4^+} f(x)$ and $\lim_{x \to 4^-} f(x)$.

2. Find $\lim_{x \to 0} x^2 sin\dfrac{1}{x}$.

3. State the Intermediate Value Theorem.

4. Find the critical numbers of $f(x) = 2x^2 + 4x$.

5. Define horizontal asymptote and vertical asymptote.

6. What is an antiderivative of a function f? Give an example.

7. Evaluate $\displaystyle\int_{-2}^{2} \dfrac{sinx}{\sqrt{1 + x^2}} dx$.

8. Define the average value of a function over an interval $[a, b]$.

9. Evaluate $\displaystyle\sum_{k=1}^{10}(2k + 1)$.

10. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on $[0, 2]$ about the x-axis.

11. State the Laws of Exponents.

12. Solve $ln(2x + 5) = 4$.

13. Draw the graph of $y = a^x$.

14. Define $sinhx$ and $coshx$.

15. Evaluate $\lim\limits_{x \to 1} \dfrac{\ln x}{x-1}$.

SECTION B: Answer the following questions. Each carries 5 marks
(Ceiling 35)

16. Find the linearization of $f(x) = \sqrt{x}$ at $a = 4$, and use it to approximate the numbers $\sqrt{3.9}$ and $\sqrt{3.98}$.

17. Let $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$ Prove that $\lim\limits_{x \to 0} f(x)$ does not exist.

18. Verify the Mean Value theorem for the function $f(x) = x^3 - 2x^2$ on the interval $[-1,\ 2]$.

19. Determine the intervals where the function $f(x) = -x^3 + 3x^2 + 1$ is increasing and where it is decreasing.

20. Find the slant asymptotes of the graph of $f(x) = \dfrac{2x^2 - 3}{x - 2}$.

21. State both parts of the Fundamental theorem of Calculus.

22. Find the length of the graph $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ on the interval $[1,3]$.

23. Evaluate $\lim\limits_{x \to \infty} \dfrac{x^3}{e^{2x}}$.

SECTION C: Answer any 2 question
(2 × 10 = 20 Marks)

24. Let $f(x) = x^4 - 4x^3 + 12$.

    (a) Find the relative extrema of $f(x)$.
    (b) Determine the intervals where the function $f(x)$ is concave upward and the intervals where it is concave downward.
    (c) Find the points of inflection of $f(x)$.

25. A car moves along a straight road with velocity function

$$v(t) = t^2 + t - 6,\ 0 \leq t \leq 10$$

Where $v(t)$ is measured in feet per second.

    (a) Find the displacement of the car between $t = 1$ and $t = 4$
    (b) Find the distance covered by the car during this period of time.

26. (a) Find the area of the region bounded by the graphs of $y = x^2 + 3$, $y = x + 1$, $x = -1$ and $x = 1$.

(b) Find the area of the surface obtained by revolving the graph of $x = y^3$ on the interval $[0, 1]$ about the y -axis.

27. (a) Evaluate $\lim\limits_{x \to 0} \dfrac{x^3}{x - \tan x}$.

(b) Find $\displaystyle\int \cosh^2 3x \, \sinh 3x \, dx$