

D1BEM2301

Reg.No.....

Name:

FIRST SEMESTER B.Sc DEGREE EXAMINATION, NOVEMBER 2023
ECONOMICS & MATHEMATICS
GDMT1B01T-BASIC CALCULUS

Time : 2.5 Hours

Maximum: 80 Marks

SECTION A: Answer the following questions. Each carries 2 marks
(Ceiling 25)

1. Let $f(x) = \begin{cases} x - 1 & \text{if } x \leq 3 \\ -2x + 8 & \text{if } x > 3 \end{cases}$; Evaluate $\lim_{x \rightarrow 3^+} f(x)$ and $\lim_{x \rightarrow 3^-} f(x)$.
2. Find $\lim_{x \rightarrow \frac{\pi}{4}} (2x^2 + \cot x)$.
3. State the Squeeze Theorem.
4. Find the critical numbers of $f(x) = 2x^3 + 6x + 7$.
5. Define Inflection point.
6. What is an antiderivative of a function f ? Give an example.
7. Evaluate $\int_1^2 (x^3 - 2x^2 + 1) dx$.
8. Evaluate $\sum_{k=1}^8 (3 - k^2)$.
9. Define the Riemann sum of $f(x)$ on $[a, b]$.
10. Find the area of the region between the graph of $y = x^2 + 2$ and $y = x - 1$ and the vertical lines $x = -1$ and $x = 2$.
11. State the Laws of Logarithms.
12. Find the derivative of $y = 3^{\sqrt{x}}$.
13. Draw the graph of $y = a^x$.
14. Define the natural logarithmic function.

15. Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2 + x}$.

**SECTION B: Answer the following questions. Each carries 5 marks
(Ceiling 35)**

16. Let $f(x) = \begin{cases} kx + 1, & x \leq 2 \\ kx^2 - 3, & x > 2 \end{cases}$

Find the value of k that will make f continuous on $(-\infty, \infty)$.

17. Find the linearization of $f(x) = \sqrt{x+3}$ at $a = 1$, and use it to approximate the numbers $\sqrt{3.9}$ and $\sqrt{4.1}$.

18. Show that the function $f(x) = x^3 + x + 1$ has exactly one zero in the interval $[-2, 0]$.

19. Determine the intervals where the function $f(x) = x + \frac{1}{x}$ is increasing and where it is decreasing.

20. Find the relative extrema of $f(x) = x^3 - 3x^2 - 24x + 32$ using the Second Derivative Test.

21. Find the average value of the function of $f(x) = 2x^2 - 3x$ over the interval $[-1, 2]$.

22. A solid has a circular base of radius 2. Parallel cross sections of the solid perpendicular to its base are equilateral triangles. What is the volume of the solid.

23. If $\sinh x = \frac{4}{3}$, find the values of the other hyperbolic functions at x .

**SECTION C: Answer any 2 question
(2 × 10 = 20 Marks)**

24. Sketch the graph of the function $f(x) = x^3 - 6x^2 + 9x + 2$.

25. (a) Find the value of c by the Mean Value theorem for Integrals for $f(x) = x^2 + 2x$ on the interval $[0, 1]$.

(b) State both parts of the Fundamental theorem of Calculus.

26. (a) Find the area of the surface obtained by revolving the graph of $y = \frac{1}{2}x + 2$ on the interval $[0, 2]$ about the x -axis.

(b) Find the arc length of the graph of $f(x) = -2x + 3$ from $P(-1, 5)$ to $Q(2, -1)$.

27. (a) Evaluate $\lim_{x \rightarrow 0} \frac{x^3}{x - \tan x}$.

(b) Find the derivative of $y = x^2 \operatorname{sech}^{-1} 3x$.