

FIRST SEMESTER B.Sc DEGREE EXAMINATION, NOVEMBER 2022**(Regular/Improvement/Supplementary)****MATHEMATICS****GMAT1B01T: BASIC LOGIC & CALCULUS****Time: 2 ½ Hours****Maximum Marks: 80****SECTION A: Answer the following questions. Each carries 2 marks.****(Ceiling 25 Marks)**

1. What is a compound proposition? Give an example.
2. Evaluate the Boolean expression $[\sim(a > b)] \vee [\sim(c < d)]$ where $a=2$, $b=3$, $c=5$, and $d=7$.
3. Negate the propositions where the UD = set of integers
 - a) $(\forall x) (x^2 = x)$
 - b) $(\exists x) (|x| = x)$
4. Find $\lim_{x \rightarrow 0} \frac{\tan x}{x}$
5. a) State Intermediate value theorem.
b) Let $f(x) = x^2 - x + 1$ on the $[-1, 4]$. Using the intermediate value Theorem, find the value of c such that $f(c) = 7$.
6. Find the interval on which $f(x) = x^3 - 3x^2 + 2$ is increasing and find the interval on which $f(x)$ is decreasing.
7. Define the critical number of a function. Show that zero is a critical number of the function $f(x) = x^3$.
8. Find the extreme values of the function $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.
9. State Rolle's Theorem. Verify the function $f(x) = x^2 - 4x + 3$ satisfies the hypotheses of Rolle's Theorem on the interval $[1, 3]$.
10. Find the relative extrema of $x^3 - 3x^2 - 24x + 32$ using second derivative test.
11. Find the point of inflection of $f(x) = x^4 - 4x^3 + 12$.
12. Find the $\lim_{x \rightarrow \infty} 2x^3 - x^2 + 1$ and $\lim_{x \rightarrow -\infty} 2x^3 - x^2 + 1$
13. Find $\int \frac{\sin t}{\cos^2 t} dt$.
14. Find the average value of $f(x) = 4 - x^2$ over the interval $[-1, 3]$.
15. Find the derivative of the function $F(x) = \int_{-1}^x \frac{1}{1+t^2} dt$.

(PTO)

SECTION B: Answer the following questions. Each carries 5 marks.

(Ceiling 35 Marks)

16. Show that $p \rightarrow q \equiv \sim q \rightarrow \sim p$ by constructing truth table.
17. Simplify the Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ by using laws of logic.
18. a) Show that $h(x) = |x|$ is continuous everywhere.
b) Use the result of part (a) evaluate $\lim_{x \rightarrow 1} \left| \frac{-x^2 - x + 2}{x - 1} \right|$.
19. a) Suppose that $y = 2x^3 - x + 1$. Find Δx and Δy when x changes from 3 to 3.01
b) Use the differential dy to approximate Δy when x changes from 3 to 3.01
c) Compute $\Delta y - dy$, the error in approximating Δy by dy
20. Show that the function $f(x) = x^3 + x + 1$ has exactly one zero in the interval $[-2, 0]$.
21. Let $f(x) = \frac{2x^2 - x + 1}{3x^2 + 2x - 1}$. Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ and find all horizontal asymptotes of the graph of f .
22. Find the area of the region under the graph of $f(x) = 4 - x^2$ on the interval $[-2, 1]$.
23. State and prove the Mean Value Theorem for integrals.

SECTION C: Answer any 2 questions. Each carries 10 marks.

24. a) Prove directly that the product of any two odd integer is an odd integer.
b) Prove indirectly that if the square of an integer is odd, then the integer is odd.
c) Prove that there are infinitely many prime numbers, using a proof by contradiction.
25. Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$
26. Find the dimensions of the rectangle of greatest area that has its base on the x -axis and is inscribed in the parabola $y = 9 - x^2$.
27. Use the definition of definite integral evaluate $\int_a^b x \, dx$.

(2 × 10 = 20 Marks)