

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022**(Regular/Improvement/Supplementary)****MATHEMATICS: COMPLEMENTARY COURSE FOR PHYSICS, CHEMISTRY & CS****GMAT1C01T: MATHEMATICS -1****Time: 2 Hours****Maximum Marks: 60**

**SECTION A: Answer the following questions. Each carries 2 marks.
(Ceiling 20 Marks)**

1. Define a singular matrix. Give one example of a 2×2 singular matrix.
2. Find the rank of (a) 3×3 identity matrix and (b) 4×2 zero matrix.
3. Define Characteristic root and Characteristic vector.
4. State Second Derivative Test of Concavity
5. Find the area of the region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x - axis.
6. State (a) Rolle's Theorem and (b) Mean Value Theorem.
7. Find $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$.
8. Evaluate $\int_0^2 \left(\frac{t^2}{4} - 7t + 5 \right) dt$.
9. Define (a) Absolute maximum and (b) Local maximum.
10. State Cavalieri's statement.
11. Define (a) Critical point and (b) Point of inflection.
12. Write the parametric representation of a circle with radius r and centre at origin.

**SECTION B: Answer the following questions. Each carries 5 marks
(Ceiling 30 Marks)**

13. Reduce $\begin{bmatrix} 2 & 0 & 4 & 6 \\ 2 & 1 & 0 & 1 \\ 8 & 2 & 8 & 14 \end{bmatrix}$ to Normal form and find its rank.
14. Solve $3x - 2y + 3z = 8$, $2x + y - z - 1 = 0$, $4x - 3y + 2z = 4$
15. Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.
16. Find $\lim_{x \rightarrow 0} x^{\frac{1}{x}}$.
17. Find the length of the graph $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \leq x \leq 4$.

(PTO)

18. The line segment $x = 1 - y, 0 \leq y \leq 1$, is revolved about the y - axis to generate the cone.

Find its Lateral Surface Area.

19. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the centre of the cylinder. Find the volume of the wedge.

SECTION C: Answer any 1 question. Each carries 10 marks.

20. (a) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

(b) Find $\frac{dy}{dx}$, if $y = \int_1^{x^2} \cos t \, dt$

21. (a) Find the critical points of $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$. Identify the intervals on which f is increasing and decreasing. Find the functions local and absolute extreme values.

(b) Find the asymptote of the curve $y = \frac{x+3}{x+2}$

(1 × 10 = 10 Marks)