S7

D1BHM2202 (PAGES 3) Reg.No............................

Name: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

HONOURS IN MATHEMATICS

GMAH1B02T: CALCULUS I

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer all the questions. Each carries 1 mark

1. If c is a real number, then $\lim_{x \to a} c =$ _________
2. If $f'(x) = g'(x)$ then $f(x) - g(x) = ?$
3. Suppose f is a differentiable function, if $f'(x) < 0$ for all x in (a, b) then f is _________ (increasing / decreasing)
4. Describe inflection points.
5. Derivative of the function $G(x) = \sqrt{1 + x^2}$ is ________
6. Value of $\int_0^{\pi/2} (cosx) dx$ is ________
7. Given that $\int_{-2}^{2} f(x)\,dx = 3$ and $\int_0^2 f(x)\,dx = 2$, evaluate the following
$$\int_2^0 2f(x)\,dx$$
8. Area of the region bounded by the graphs of y=x, x=0 and y=3 is _________
9. Equation to find the volume of a solid with known cross section is __________
10. The arc length formula is _________

(10 x 1 = 10 Marks)

PART B: Answer any eight questions. Each carries 2 marks

11. Explain what is meant by the statement $\lim_{x \to 3^-} f(x) = 2$.
12. Find the values of x for which the function is continuous
$$f(x) = x^8 - x^4 + x + 4 + \frac{(x+1)}{(x+1)(x-2)}$$
13. Is $x = \frac{\pi}{2}$ an asymptote of $f(x) = \tan x$
14. Find the horizontal asymptote of $f(x) = \frac{2x}{\sqrt{x^2 - 1}}$
15. Define the term 'Solution of a Differential Equation' with example.

(PTO)

16. Find the function f given that the slop of the tangent line to the graph of f at any point (x, f'(x)) is $x^2 - 2x + 3$ and the graph of f passes through the point (1,2).

17. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

18. By revolving region under the graph of $y = \sqrt{r^2 - x^2}$ on [-r, r], show that the volume of the sphere of radius r is $V = \frac{4}{3}\pi r^3$.

19. Use differentials to obtain an approximation of the arc length of the graph of $y = 2x^2 + x$ from P(1, 3) to Q(1.1, 3.52).

20. Define surface revolution

(8 x 2 = 16 Marks)

PART C: Answer any six questions. Each carries 4 marks.

21. Find the relative extrema of $f(x) = x^4 - 4x^3 + 12$ using first derivative test.

22. Find the points of inflection $f(x) = (x-1)^{\frac{1}{3}}$

23. Find the asymptote of the function $f(x) = \frac{1}{x-1}$

24. A model rocket is fired vertically upward from a height of s_0 ft above the ground with a velocity v_0 ft/sec. If air resistance is negligible, show that its height (in feet) after t seconds is given by

$$s(t) = -16t^2 + v_0 t + s_0$$

(g = 32ft/sec)

25. A car moves along a straight road with velocity function

$$v(t) = t^2 + t - 6 \qquad 0 \le t \le 10$$

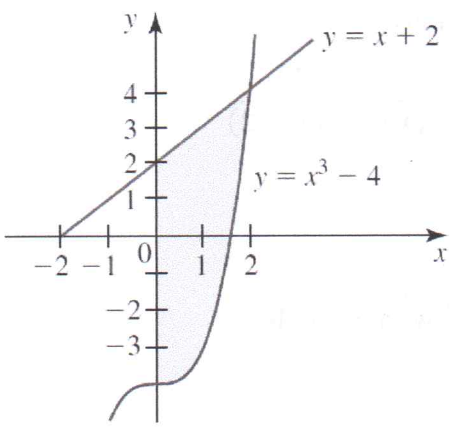
where *v(t)* is measured in meter per second.

a. Find the displacement of the car.

b. Find the distance covered by the car during this period of time.

26. Evaluate $\int_0^{\pi/2} \sec x \, dx$ by finding the integral of the function *secx* using substitution method

27. Find the area of the shaded region



28. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated axis.

$y = -x^2 + 2x, y = 0$and the x axis

(6 x 4 = 24 Marks)

PART D: Answer any two questions. Each carries 15 marks.

29. a. Find the linearization of $f(x) = \sqrt{x}$ at a = 4

b. Plot an approximate graph

c. Use the result of part (a) to approximate the numbers $\sqrt{3.9}$, $\sqrt{3 \cdot 98}, \sqrt{4}, \sqrt{4.04}$, $\sqrt{4 \cdot 8}, \sqrt{6}$ and $\sqrt{8}$. Compare the results with the actual values.

30. Find the extreme values of the function. $f(x) = 2cosx - x$on $[0,2\pi]$.

31. The speed of a cyclist is measured at 4-sec intervals over a 32-sec time span and recorded in the following table.

Time (s)	0	4	6	12	16	20	24	28	32
Speed (ft/sec)	2	4	6	10	12	14	10	8	6

If we let denote the velocity function associated with the motion of the cyclist over the time interval [0, 32], then the values of are available to us only at a discrete set of numbers, even though *v* is clearly a continuous function defined on the interval. Find the approximate distance *D* covered by the cyclist from *t=0* to *t=32* using

a. Eight rectangles and choosing c_k to be the left endpoint of the n^{th} subinterval

b. Eight rectangles and choosing c_k to be the right endpoint of the n^{th} subinterval

c. Four rectangles and choosing c_k to be the midpoint of the n^{th} subinterval.

(2 x 15 = 30 Marks)