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D1BHM2201 (PAGES 2) Reg.No.....

Name: .....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

HONOURS IN MATHEMATICS

GMAH1B01T: THEORY OF EQUATIONS AND COMPLEX NUMBERS

Time: 3 Hours **Maximum Marks: 80**

PART A: Answer all the questions. Each carries 1 mark.

Choose the Correct Answer

1. If $f(x) = 3x^3 - 2x^2 + 3x + 4$, then $f(-2)$ *is* .....
   a) -34 b) 34 c) -30 d) 36
2. One possible integral root of $x^5 - 6x^4 - 5x^3 + 8x^2 + 3x - 24 = 0$ is.....
   a) 10 b) 4 c) 5 d) None of the above
3. One possible integral root of $x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$ is.....
   a) 3 b) 4 c) 5 d) None of the above
4. For small positive values of x, the equation $-2x^3 + 3x^5 - 100x^6$ is.....
   a) Positive b) Negative c) Either positive or negative d) Zero
5. The number of permanences in the sequence $4, 4, -2, -3, 4, 4, -1, 7, 7, 8, -5, -6, -7$ is...
   a) 5 b) 6 c) 4 d) None of the above

Fill in the Blanks

6. If $f(x) = 4x^3 + x^2 + 2x - 3$, then $f(2)$ *is* ----------
7. The remainder when dividing $f(x) = x^3 + x^2 - 5x + 3$ by $x - 3$ is ----------
8. The method to solve biquadratic equation is ----------
9. The number of variations in the sequence

   $1, -2, -3, 4, -4, -1, 7, 7, 8, -5, -6, -7$ is ----------
10. $(3 + 2i)(-2 + i) =$ ----------

(10 x 1 = 10 Marks)

PART B: Answer any 8 questions. Each carries 2 marks

11. Show that $f(x) = 2x^3 - 3x^2 - 5x - 12$ is divisible by $x - 3$.
12. Show that $x^n - c^n$ is divisible by $x - c$.
13. Give Taylor's formula.
14. Find a lower limit of the negative roots of the equation.

$$3x^5 - 8x^4 - 10x^3 + 6x^2 + 3x - 10 = 0$$

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15. Find an upper limit of the positive roots of the equation.

$$3x^5 - 8x^4 - 10x^3 + 6x^2 + 3x - 10 = 0$$

16. Give the cubic resolvent in Ferrari's method to solve a biquadratic equation.
17. Verify that the equation $2x^4 - 7x + 2 = 0$ has a root in the interval $(1,2)$.
18. State Rolle's Theorem.
19. Show that $1 + i$ satisfies the equation $z^2 - 2z + 2 = 0$.
20. Define *boundary point* of a set.

(8 x 2 = 16 Marks)

PART C: Answer any 6 questions. Each carries 4 marks.

21. Write a polynomial of the lowest degree that for $x = 0$ takes the value 3 and has the following roots: 2 and -1 as simple roots, -2 as a double root, 3 as a triple root
22. Factorize $x^3 - 1$ into linear factors.
23. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$
24. Solve the equation $x^3 + 9x^2 + 6x - 56 = 0$, roots are a, b, c and $b = -2a$.
25. Find limits of the roots for the equation $6x^5 - 27x^4 - 100x^3 - 200x - 50 = 0$
26. Show that for all real values of λ the equation.

$$(x-2)(x-5)(x-7)(x-9) + \lambda(x-3)(x-6)(x-8)(x-10) = 0$$

has all roots real and simple and separate them.

27. Give the exponential form of $-1 - i$.
28. Find $\frac{4+2i}{-2+3i} + \frac{2+i}{3i}$

(6 x 4 = 24 Marks)

PART D: Answer any 2 questions. Each carries 15 marks.

29. Using Cardan's formula, solve $x^3 - 15x^2 + 105x - 245 = 0$
30. How many real roots do the following equations have?
    a) $f(x) = x^5 + x^3 - 2x^2 + x - 2 = 0$
    b) $(x) = x^5 + 2x^3 - x^2 + x - 1 = 0$
    c) $(x) = 1 - 2x + 3x^2 - 4x^3 + 5x^4 = 0$
31. a) By writing the individual factors on the left in exponential form, performing the needed operations and finally changing back to rectangular coordinates, show that $(-1 + i)^7 = -8(1 + i)$

    b) Find all values of $(-16)^{\frac{1}{4}}$

(2 x 15 = 30 Marks)