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D1BEM2202 (PAGES 2) Reg.No............................

Name: ....

FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT1B01T: BASIC CALCULUS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries 2 marks. (Ceiling 25 Marks)

1. Define right hand limit of a function.
2. Define absolute maximum and absolute minimum of a function f.
3. Find the extrema of a function, if any, by examining its graph.
$$f(x) = x^2, -1 \leq x \leq 2$$
4. State Mean Value Theorem.
5. Find the point of inflection of $f(x) = x^4 - 4x^3 + 12$
6. Evaluate $\int \frac{1}{x^3} dx$.
7. Find $\int (2x + 3\sin x)dx$.
8. Evaluate the sum.
$$\sum_{k=1}^{50} k^2$$
9. Define a partition of an interval [a,b] and the norm of a partition.
10. Write an integral giving the arc length of the graph of the function $y = \cos x$ on the interval $[0, \pi]$.
11. Define the arc length function for the graph of a smooth function $f(x)$.
12. Let f be a non-negative smooth function on [a, b]. What is the surface area of the surface obtained by revolving the graph of f about the x-axis.
13. Let f be smooth on [a, b] . Then write the formula for finding the arc length of the graph f from (a, f(a)) to (b, f(b)).
14. Evaluate the expression $\log_3 \frac{1}{81}$
15. Prove the identity $\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$.

(PTO)

SECTION B: Answer the following questions. Each carries 5 marks.
(Ceiling 35 Marks)

16. Show that $\lim_{x \to 2} [\![x]\!]$ does not exist, where $[\![x]\!]$ denote the greatest integer function.
17. Find the vertical asymptotes of the graph of $f(x) = \frac{x}{x^2-x-2}$
18. Determine the intervals where the function $f(x) = x^3 - 3x^2 + 2$ is increasing and where it is decreasing.
19. Compute the Riemann sum for $f(x) = 4 - x^2$ on [-1,3] using five subintervals (n= 5) and choosing the evaluation points to be the midpoints of the subintervals.
20. Differentiate the functions a) $y = \sqrt{\cosh^{-1} 2x}$ b) $y = \frac{\cosh^{-1} t}{1+\tanh 2t}$
21. Evaluate (a) $\lim_{x \to \infty} \frac{\ln(1+e^x)}{x^2}$ (b) $\lim_{x \to 0} (\cos x - x)^n$
22. Find the derivative of a) $g(t) = \ln\left|\frac{\sin t+1}{\cos t+2}\right|$ b) $g(x) = \ln\sqrt{\frac{x\cos x}{(2x+1)^3}}$
23. Use logarithmic differentiation to find the derivative of the function $y = (x^2 + x)^x$

SECTION C: Answer any 2 questions. Each carries 10 marks.

24. Let $f(x) = \begin{cases} 1 & if\ x \geq 0 \\ -1 & if\ x < 0 \end{cases}$. Prove that $\lim_{x \to 0} f(x)$does not exist.
25. A man has 100 ft of fencing to enclose a rectangular garden in his backyard. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.
26. State and prove the mean value theorem for integrals.
27. By revolving the region under the graph of $y = \sqrt{r^2 - x^2}$on $[-r, r]$, show that the volume of a sphere of radius r is $V = 4\pi r^3$

(2 x 10 = 20 Marks)