D4AST2302 (2 Pages) Name............................
Reg.No..........................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025
(Regular/Improvement/Supplementary)

STATISTICS
FMST4E13 - STATISTICAL DECISION THEORY & BAYESIAN ANALYSIS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define frequentist risk of decision rule. When do you say that a decision rule is admissible? Illustrate this concept with the help of an example.
2. Differentiate between randomized and non-randomized decision rules. Explain point estimation and test of hypothesis as decision rules.
3. Discuss any two loss functions associated with statistical decision theory.
4. Show that Bayes rule when it is unique is admissible
5. Discuss relative likelihood approach for prior selection.
6. What is the need of a non-informative prior? When it will become improper?
7. Explain minimax theorem for game with finite parameter space and arbitrary action space.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Discuss various steps in the construction of Utility function.
9. Let x have a Poisson distribution with parameter θ. Consider the problem of point estimation with squared error loss function. If the prior distribution is $\pi(\theta) = e^{-\theta}$ and the class of decision rule is $\delta_c(x) = cx$, find the frequentist risk. Show that $\delta_c(x)$ is inadmissible when c > 1. Also find the Bayes risk.
10. Define Jeffrey's non informative. Determine Jeffrey's non informative prior when x follows a Poisson distribution with parameter θ.
11. Discuss the role of marginal distribution in determining prior.
12. Formulate interval estimation and test of hypothesis from a Bayesian point of view.
13. Suppose x1, x2, …, xn be a sample from N (θ, σ^2), σ^2 known. Show that under squared error loss $\bar{x}$ is minimax.
14. Write a note on empirical Bayes procedure.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Define the notion of entropy. How it can be used to obtain an appropriate prior in the presence of partial information. Illustrate it with the help of an example.
16. Let x1, x2,…,xn be a sample from U(0, θ) and θ has prior distribution $\pi(\theta) = \frac{\alpha\theta_0^\alpha}{\theta^{\alpha+1}}$, $\theta > \theta_0$. where θ_0 and α are known. Find Bayes estimate of θ under squared error loss. Also find posterior variance.
17. Let x1, x2,…,xn are the observations from N (θ, σ^2), σ^2 known. If L (θ, a) = $(\theta - a)^2$, show that $\bar{x}$ is admissible.
18. Consider the simple normal regression model:

    Yi = β xi + εi, i =1,2,…,n, where εi ~ N (θ, σ^2), σ^2 known. Also εi's are independent.

    Find posterior distribution of β when:

    i) the prior distribution is $\pi(\beta) = 1$ ii) Conjugate prior for β.

(2 × 5 = 10 weightage)