D4AMT2302 **Reg.No.........................**

Name: ....

FOURTH SEMESTER M. Sc. DEGREE EXAMINATION, APRIL 2025
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH4E08 - ALGEBRAIC TOPOLOGY

Time : 3 Hours **Maximum Weightage: 30**

Part A

Answer *all* questions. Each carries 1 weightage.

1. Give geometrically independent set of four points in $\mathbb{R}^3$.
2. Let K be the closure of a 2-simplex $\sigma = \langle a_0a_1a_2 \rangle$. List all elements of K.
3. Let z_p be a p-cycle on a complex K. Prove that the homology class $[z_p]$ and the coset $z_p + B_p(K)$ are identical.
4. Define barycentric subdivision of CI(σ^r).
5. State the Hopf Classification theorem.
6. Verify whether the n-sphere is a contractible space for $n \geq 0$.
7. Let $\alpha(t) = t$ and $\beta(t) = t^2$ be paths in $\mathbb{R}$. Give a homotopy between α and β.
8. When a path connected space is simply connected ? Give an example.

(8 × 1 = 8 weightage)

Part B

Answer *any two* questions from *each unit.* Each carries 2 weightage.

Unit I

9. Describe the traingulation of the Mobious strip.
10. Show that every simplex is a convex set.
11. Define the p^{th} incidence matrix of an oriented geometric complex K. Compute the 2nd incidence matrix of the closure of the 3-simplex $\langle a_0a_1a_2a_3 \rangle$ with vertices ordered by $a_0 < a_1 < a_2 < a_3$.

Unit II

12. Verify which of the points $(\frac{1}{2}, \frac{1}{2}, 0)$, $(\frac{1}{3}, \frac{1}{3}, 0)$ belong to the interior of 2-simplex with vertices (0, 0, 0), (1, 0, 0) and (0, 1, 0).

13. Prove that an n-pseudomanifold L is nonorientable if and only if $H_n(L) = 0$.

14. Prove that there is a vector field on S^n , $n \geq 1$ if and only if n is odd.

Unit III

15. Prove that homotopy of paths is an equivalence relation on the set of all paths in a topological space X.

16. Prove that every contractible space is simple connected.

17. If A is a deformation retract of a space X and x_0 is a point of A, then prove that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(A, x_0)$.

($6 \times 2 = 12$ weightage)

Part C

Answer *any two* questions. Each carries 5 weightage.

18. (a) When do you say that two simplexes are connected?

    (b) If a polyhedron $\mid K \mid$ has r components and tringulation K, then prove that $H_0(K)$ is isomorphic to the direct sum of r copies of $\mathbb{Z}$.

19. (a) Let K be an oriented geometric complex of dimension n, and for $p = 0, 1, \ldots.., n$ let α_p denote the number of p-simplexes of K. Then prove that

$$\sum_{p=0}^{n}(-1)^p\alpha_p = \sum_{p=0}^{n}(-1)^pR_p(K)$$

    where $R_p(K)$ denotes the p^{th} Betti number of K.

    (b) Let K be a 2-pseudomanifold with α_0 vertices, α_1 1-simplexes, and α_2 2-simplexes. Then Prove that

    i. $3\alpha_2 = 2\alpha_1$,

    ii. $\alpha_1 = 3(\alpha_0 - \chi(K))$,

    iii. $\alpha_0 \geq \frac{1}{2}(7 + \sqrt{49 - 24\chi(K)})$.

20. (a) Define chain mapping and simplicial mapping on a complex K. If $\phi : K \to L$ is a simplicial mapping, then show that the sequence $\{\phi_p\}_0^\infty$ of homomorphisms induced by ϕ is a chain mapping.

    (b) Define mesh of a complex K. Prove that $\lim\limits_{s\to\infty} meshK^{(s)} = 0$ for any complex K.

21. (a) Prove that two loops α and β in S^1 with base point 1 are equivalent if and only they have the same degree.

    (b) Show that the fundamental group $\pi_1(S^1)$ is isomorphic to the group $\mathbb{Z}$ of integers under addition.

($2 \times 5 = 10$ weightage)