D4AST2202 **(2 Pages)** **Name..............................**
Reg.No...........................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)

STATISTICS
FMST4E13 - STATISTICAL DECISION THEORY & BAYESIAN ANALYSIS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Explain briefly the utility function.
2. Define loss function. Explain any two loss functions used in statistical decision theory.
3. What are conjugate priors? Discuss the use of such priors.
4. Explain the histogram approach of subjective determination of prior density.
5. Show that Bayes rule when it is unique, is admissible.
6. Prove that under squared error loss function mean of the posterior distribution is the Bayes estimator.
7. Write a note on empirical Bayes estimators.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Let X have a Poisson distribution with mean θ. Let A= [0,∞), be the action space. The loss function is $(\theta, a) = (\theta - a)^2$. Consider the decision rules of the form $\delta_c(x) = cx. Assume\ \pi(\theta) = e^{-\theta}$ is the prior density. Find $\rho(\pi, a)$ and Bayes action.
9. Discuss various steps in the construction of utility of money.
10. Describe ML –II approach to prior selection.
11. State minimax theorem and explain randomized strategies in game theory.
12. Explain what is meant by predictive inference. How it is handled in classical inference?
13. Formulate interval estimation and testing problem from a Bayesian point of view.
14. Let $X = (X_{1,} X_{2,} \cdots X_n)$ be a sample from Poisson (θ) and suppose $\theta \sim Gamma\ (\alpha, \beta)$. If X_{n+1} is a future observation, find predictive pmf of X_{n+1} given $X_{1,} X_{2,} \cdots X_n$.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Let $X_{1,}\ X_{2,\ldots} X_n$ be iid $N(\theta, 1)$. We want to estimate θ under $L(\theta, a) = (a - \theta)^2$. Show that $\overline{X_n} = \frac{1}{n} \sum X_i$ is admissible.

16. What is the need of non-informative prior? When it will become improper? Discuss the derivation of non-informative invariant priors for location and scale families.

17. Let $X_{1,}\ X_{2,\ldots} X_n$ be a sample from U(0,θ). Let θ has prior distribution $\pi(\theta) = \frac{\alpha {\theta_0}^{\alpha}}{\theta^{\alpha+1}}, \theta > \theta_0$ where θ_0 and α are known.

    (a) Find posterior distribution of θ.

    (b) Determine Bayes estimate of θ under squared error loss function.

18. Let X= $(X_{1,}\ X_{2,\ldots} X_n)$ be random sample from$N(\theta, r)$, θ and precision $r = \frac{1}{\sigma^2}$ both unknown. If $\pi\ (\theta, r) \propto \frac{1}{r}$, find posterior density of X_{n+1} given X= $(X_{1,}\ X_{2,\ldots} X_n)$.

(2 × 5 = 10 weightage)