D4AST2002 **(2 Pages)** Reg. No: ....

Name: ....

FOURTH SEMESTER M.Sc DEGREE EXAMINATION,APRIL 2022

(Regular/Improvement/Supplementary)

STATISTICS

FMST4E13: STATISTICAL DECISION THEORY AND BAYESIAN ANALYSIS

Time : 3 Hours. **Maximum Weghtage : 30.**

Part A: Answer any four questions. Each carries two weightage.

1. Differentiate randomized and non-randomized decision rules with examples.
2. Discuss the notion of preference pattern in utility theory.
3. Explain the minimax principle.
4. If $f(x|\theta)$ is $B(n,\theta)$ density and prior is $\pi(\theta) \sim beta(\alpha,\beta)$. Find the marginal density of X and hence the posterior density of $\theta|X$.
5. Give the explanation of Laplace rule of succession.
6. State Lindley's procedure for test of significance.
7. Define absolute error loss function. What is the Bayes estimator under this loss function.

($4 \times 2 = 8$ weightage)

PART B: Answer any four questions. Each carries three weightage.

8. Discuss the axiomatic development of utility and explain how utility function is constructed.
9. Determine the Jeffrey's non-informative prior for the unknown vector of parameters in Gamma(α,β) distribution.
10. Prove that a unique minimax strategy is admissible.
11. Let $X \sim U(0,\theta)$. Suppose that the prior density $\pi(\theta) = \frac{\alpha}{\theta^{\alpha+1}}; \theta \geq 1$. Using the quadratic loss function, find the Bayes estimator of θ.
12. Let $X \sim B(n,\theta)$. The prior density of θ is $U(0,1)$. Find Bayes estimator of θ using loss function $L(\theta,T) = \frac{(\theta-T)^2}{\theta(1-\theta)}$.

13. Write a note on general linear model.

14. Explain the concept of predictive inference.

(4 × 3 =12 weightage)

PART C: Answer any two questions. Each carries five weightage.

15. a) Discuss the basic elements of a statistical decision problem and illustrate them with a suitable example.

    b) Define 0-1 loss function and mention its use in testing situations.

16. a) Show that if $L(\theta, a) = w(\theta)(\theta - a)^2$, the Bayes rule is

$$\delta^*(x) = \frac{\int \theta w(\theta) f(x|\theta) d\pi(\theta)}{\int w(\theta) f(x|\theta) d\pi(\theta)}.$$

    b) Let $X_1, X_2, \cdots, X_n$ be iid $U(0, \theta)$ random variables. Suppose the prior distribution is

$$\pi(\theta) = \begin{cases} \frac{\alpha\beta^\alpha}{\theta^{\alpha+1}}, & \theta \geq \beta, \alpha > 0 \\ \theta, & \theta < \beta. \end{cases}$$

    If $L(\theta, \delta) = (\theta - \delta)^2$, find Bayes estimator of θ.

17. a) What is meant by non-informative prior?. Give examples.

    b) Define improper priors and maximum entropy priors.

    c) Prove that if an equalizer strategy is admissible, then it is minimax.

18. a) Distinguish between homoscedastic disturbances and hetroscedastic disturbances.

    b) Explain the concept of robustness of Bayes rules.

(2 × 5 =10 weightage)