

**FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022**  
**(Improvement/Supplementary- 2019 Admission)**

**STATISTICS**  
**FMST4E11 - TIMESERIES ANALYSIS**

**Time: 3 Hours**

**Maximum Weightage: 30**

**Part A: Answer any *four* questions. Each carries *two* weightage.**

1. Define spectral density  $f(\lambda)$  of a time series. Find the spectral density of a first order moving average (MA(1)) model.
2. Obtain the Yule-Walker equation satisfied by the ACF of an autoregressive process of order  $p$ (AR( $p$ )) model.
3. Define a time series and explain the additive and multiplicative models of time series.
4. Describe the role of residual analysis in time series.
5. Define periodogram of a time series and mention its applications.
6. Compute the autocorrelation function (ACF) of a process  $X_t = Z_t + \theta Z_{t-1}$  where  $Z_t$ 's are independent and identically distributed (i.i.d) $N(0,1)$  random variables.
7. Explain the difference between autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models, clearly mentioning the need for ARIMA models.

**(4 × 2 = 8 weightage)**

**Part B: Answer any *four* questions. Each carries *three* weightage.**

8. Distinguish between weak and strong stationarity in time series. Check the stationarity of the time series  $X_t = (-1)^t Y$ , where  $Y$  is a random variable with mean 0 and variance 4.
9. Prove that the ACF is even and it lies between -1 and +1.
10. Establish the duality between MA process and AR(1) process. Also prove that an AR(1) process is Markovian.
11. Derive the stationarity conditions of an AR(2) model.
12. Derive an 1-step ahead forecasting formula for ARIMA(2,1,1) model using difference equation form.
13. Discuss on the least square estimation of AR(1) and MA(1) models.
14. Define GARCH model. State the conditions for its stationarity. Highlight its application in time series analysis.

**(4 × 3 = 12 weightage)**

**(P.T.O.)**

**Part C: Answer any two questions. Each carries five weightage.**

15. (a) Let  $\{e_t\}$  be a zero mean white noise process. If  $Y_t = e_t + \theta e_{t-1}$ , find the autocorrelation function for  $\{Y_t\}$ , 1) when  $\theta = 2$  ; 2) when  $\theta = \frac{1}{2}$ .

(b) Describe the method of determining the order of AR and MA parameters in time series analysis.

16. Explain the Holt method and Holt Winter method (additive and multiplicative cases) of smoothing techniques in time series.

17. (a) State and prove Herglotz theorem.

(b) Define an ARCH(1,1) model and state its important properties.

18. (a) Differentiate between ARMA and ARIMA model.

(b) Explain in detail Box-Jenkin's approach to time series analysis.

**(2 × 5 = 10 weightage)**