(3 Pages)

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Name:....................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

STATISTICS

FMST4C13: MULTIVARIATE ANALYSIS

Time: 3 Hours Max. Weightage: 30

Part A: Answer *any four* questions. Each carries *two* weightage

1. Let X_1, X_2, X_3 be independent univariate normal random variables with $E(X_1) = E(2X_2) = E(3X_3) = 1$ and $V(X_1) = V(X_2) = V(X_3) = 1$. Define $Y_1 = X_1$, $Y_2 = 2X_2$, $Y_3 = 3X_3$. If $\underset{\sim}{Y} = (Y_1, Y_2, Y_3)'$, identify the distribution of $\underset{\sim}{Y}$.

2. If $\underset{\sim}{X} \sim N_p(0, \Sigma)$, where Σ is a positive definite matrix, derive the distribution of $X'\Sigma^{-1}X$.

3. Describe how do you test the hypothesis $H_0 : \underset{\sim}{\mu} = \underset{\sim}{\mu_0}$, a given vector in $\Re^p$, given a random sample of size N from $N_p(\mu, \Sigma)$ when the dispersion matrix Σ is known.

4. Define Mahalanobis D^2-statistic. Describe any two uses of D^2-statistic.

5. What is spherical normal distribution? What hypothesis you test in sphericity test?

6. Describe classification problem. Illustrate it through an example.

7. Describe Wishart distribution. Is it related to any of the univariate distributions? Justify your answer.

(4 x 2= 8 weightage)

Part B: Answer *any four* questions. Each carries *three* weightage

8. Prove that a random vector $\underset{\sim}{X}$ has multivariate normal distribution if and only if every linear combination $l'x$ is univariate normal.

9. Explain the following terms:

   (i) Factor loadings (ii) Orthogonal factor model.

10. State the necessary and sufficient condition for the independence of linear form in $\underset{\sim}{X}$ and a quadratic form in $\underset{\sim}{X}$, when $\underset{\sim}{X} \sim N_p(0, \Sigma)$. Using this result verify that sample mean $\bar{X}$ and sample variance S^2 based on a sample from univariate normal distribution are independent.

11. Given $\{X_\alpha, \alpha = 1, 2, \ldots, N\}$ be a random sample from $N_p(\mu, \Sigma)$ where μ is a known vector. Derive the maximum likelihood estimator of Σ.

12. Write down the characteristic function of Wishart distribution. State and prove the additive property of Wishart distribution.

13. Given a random sample from $N_p(\mu, \Sigma)$ where Σ is unknown, prove that the statistic to be used for testing $H_0 : \mu = \mu_0$, a given vector in $\Re^p$, is the Hotelling's T^2 statistic.

14. Describe how do you classify an observation into one of two known multivariate normal populations.

(4 x 3= 12 weightage)

Part C: Answer *any two* questions. Each carries *five* weightage

15. a) Let $X \sim N_p(\mu, \Sigma)$ and $Y = CX$, where C is a non-singular matrix of order $p \times p$. Prove that $Y \sim N_p(C\mu, C\Sigma C')$.

    b) Suppose $\underset{\sim}{X} \sim N_3(0, \Sigma)$ where $\Sigma = \begin{pmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}$. If $\underset{\sim}{X}$ is partitioned into two subvector $X^{(1)} = \begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$, $X^{(2)} = X_2$, find the conditional distribution of $X^{(1)}$ given $X^{(2)} = x_2$.

16. a) Derive the likelihood ratio test for testing $H_0 : \Sigma = \Sigma_0$ based on a random sample from $N_p(\mu, \Sigma)$ when μ is unknown.

    b) Derive the sphericity test.

17. a) Write down the maximum likelihood estimators of the parameters of the multinormal distribution. Check whether the estimators are (i) unbiased and (ii) consistent for the parameters.

b) Let $A \sim W_p(n, \Sigma)$ and with usual notation A is partitioned as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}.$$

Derive the marginal distribution of A_{11}.

18. a) Describe how do you classify an observation into one of two multinormal populations when the population parameters are unknown.

b) Derive the first two principal components of $\underset{\sim}{X}$ when $\underset{\sim}{X} \sim N_3(0, \Sigma)$ where $\Sigma = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 5 & -3 \\ 3 & -3 & 5 \end{pmatrix}$.

(2 x 5= 10 weightage)