D3AST2401 **(2 Pages)** **Name.............................**
Reg. No..........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)

STATISTICS
FMST3C11 - STOCHASTIC PROCESSES

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Describe the classification of stochastic processes based on their index set and the state space.
2. Define a Markov Process with an example.
3. Define the inter-arrival time distribution in a Poisson process. Show that it follows the exponential distribution.
4. If r_t and δ_t are independent for all $t \geq 0$, show that the process is a Poisson process.
5. Find a necessary and sufficient condition for a pure birth process to converge.
6. Define Brownian motion process. State any two important properties.
7. State the basic characteristics of a queueing system.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Classify the states of the Markov chain having tpm given by

$$P=\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

9. Define a random walk. Discuss its applications.
10. Write short notes on the following.
    i) Non-homogeneous Poisson process.
    ii) Compound Poisson process.
    iii) Conditional mixed Poisson process.
11. Define the renewal function M(t) and show that it satisfies the integral equation $M(t) = F(t) + \int_0^t M(t-y)\, dF(y),\ t \geq 0.$
12. Define a birth and death process. Derive the differential equations governing its transition probabilities.

(P.T.O.)

13. Explain the characteristics of an M/M/1 queueing system. Derive the mean number of customers in the system.

14. Distinguish between stationary and weakly stationary processes, citing one example to each.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. i) Derive Chapman-Kolmogorov Equations.

    ii) Describe Gambler's ruin problem. Find the probability of ultimate ruin of the gambler.

16. State and prove any two important properties of a homogeneous Poisson process. Use them to derive the distribution of the number of arrivals in a time interval of length 't'.

17. What is a semi-Markov process? Compare it with continuous-time Markov chains and regenerative processes, giving suitable examples.

18. Compare and contrast the G/M/1 and M/G/1 queueing models with suitable examples.

(2 × 5 = 10 weightage)