(2 Pages)

D3AMT2405

Reg. No.....
Name: .....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3E01 - CODING THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. Define Parity Check digit. Modify $C = \{000, 011, 101, 110\}$ by adding a 4th parity digit.
2. Calculate $\Phi_{.97}(10110, 01001) = ...$
3. Find the Dual code $C^{\perp}$, where $C =< S >,\ S = \{0101, 1010, 1100\}$.
4. Find an upper bound for the dimension of a linear code with $n = 10,\ d = 5$.
5. Does there exists perfect codes for $n = 15,\ d = 3$. Justify.
6. Define the Hamming code of length $2^r - 1$ and find its dimension.
7. Let $g(x) = 1 + x^2 + x^3$ be the generator polynomial of a linear cyclic code of length 7. Encode the message polynomial $1 + x^3$.
8. Find all words in v of length 5 such that $\pi(v) = v$.

(8 x 1 = 8 weightage)

Part B: Answer *any two* questions from *each unit*. Each carries 2 weightage.

Unit I

9. a) Show that a code C of distance d will atleast detect all nonzero error patterns of weight less than or equal to $d - 1$ and there is atleast one error pattern of weight d which C will not detect.
   b) Define t-error detecting code.
10. Find a basis for $C^{\perp}$ where $C =< S >,\ S = \{11101, 10110, 01011, 11010\}$.
11. Construct an SDA assuming IMLD for $C = \{0000, 1011, 0101, 1110\}$ and hence decode $w = 1101$.

(P.T.O.)

Unit II

12. Find a generating and parity check matrix for an extended Hamming code of length 8.

13. Verify Hamming bound for linear code C with generator matrix $G = \begin{pmatrix} 100111 \\ 010101 \\ 001011 \end{pmatrix}$

14. Find generator matrix $G(2,3)$ for $RM(2,3)$ code.

Unit III

15. Find a parity check matrix for the linear cyclic code of length $n = 7$ and generator polynomial $g(x) = 1 + x + x^3$.

16. Show that every cyclic code contains a unique idempotent polynomial which generates the code.

17. Find the generator polynomial for the dual cyclic code of length 6 with generator polynomial $g(x) = 1 + x + x^2$.

(6 x 2 = 12 weightage)

Part C: Answer *any two* questions. Each carries 5 weightage.

18. a) Define Parity Check matrix for a linear code.

    b) Let $H = \begin{pmatrix} 11 \\ 11 \\ 01 \\ 10 \\ 01 \end{pmatrix}$ be the parity check matrix of C.

    Find parity check matrix of $C^{\perp}$.

19. Show that the Distance of C_{24} is 8.

20. Decode $w = 101011100000,\ 10101011011$ which is encoded using C_{23}.

21. Let $n = 15$ and generator polynomial for a linear cyclic code C with $d = 5$ be $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. Decode $w = 110011100111000$ which is encoded using C.

(2 x 5 = 10 weightage)