(3 Pages)

D3AMT2404 **Reg.No.........................**

Name: ....

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3C14-PDE AND INTEGRAL EQUATIONS

Time : 3 Hours **Maximum Weightage: 30**

Part A

Answer *all* questions. Each carries 1 weightage.

1. Solve $u_x + u_y = 2$ subject to the condition $u(x, 0) = x^2$.
2. Describe Domain of dependence and region of influence.
3. Let $u(x, t)$ be a solution for wave equation $u_{tt} - u_{xx} = 0$ which is defined in the whole plane. Assume that $u_x(x, t)$ is constant on the line $x = 1 + t$, $u(x, 0) = 1$ and $u(1, 1) = 3$. Find such a solution $u(x, t)$.
4. Show that the only possible value for the eigen value problem $\frac{d^2X}{dx^2} + \lambda X = 0$, $0 < x < L$, $X(0) = X(L) = 0$ are positive real numbers.
5. Show that the Dirichlet problem has atmost one solution in a smooth domain.
6. State the strong maximum principle of harmonic functions.
7. Explain the volterra equation of third kind? Can the same be transformed to that of second kind ? Justify.
8. Find the resolvent kernal for $k(x, t) = xe^t$ on $(-1, 1)$.

(8 × 1 = 8 weightage)

Part B

Answer *any two* questions from *each unit.* Each carries 2 weightage.

Unit I

9. Solve the equation $u_x + u_y + u = 1$, subject to the initial condition $u = sinx$, on $y = x + x^2$, $x > 0$.
10. Find the canonical form of $u_{xx} + 6u_{xy} - 16u_{yy} = 0$ and find the general solution.
11. Explain and justify the well posedness of Cauchy problem.

(P.T.O.)

Unit II

12. Solve $u_t - u_{xx} = 0$, $0 < x < \pi$, $t > 0$,
    $u(0,t) = u(\pi,t) = 0$, $t \geq 0$,
    $$u(x,0) = f(x) = \begin{cases} x & 0 \leq x \leq \frac{\pi}{2}, \\ \pi - x & \frac{\pi}{2} \leq x \leq \pi. \end{cases}$$

13. Show that the Neumann problem for the vibrating string
    $u_{tt} - c^2 u_{xx} = F(x,t), 0 < x < L,\ t > 0$ subject to the conditions
    $u_x(0,t) = a(t),\ u_x(L,t) = b(t)\ t \geq 0,$
    $u(x,0) = f(x)\ 0 \leq x \leq L,$
    $u_t(x,0) = g(x)\ 0 \leq x \leq L,$ has a unique solution.

14. Prove that the Green function for the Dirichlet problem is symmetric.

Unit III

15. Transform the problem
    $y'' + xy = 1, y(0) = 0, y(l) = 1$ to an integral equation.

16. State four properties of Green's function.

17. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real.

(6 × 2 = 12 weightage)

Part C

Answer *any two* questions. Each carries 5 weightage.

18. State and prove the existence theorem for quasilinear first order partial differential equations.

19. (a) State and prove the mean value principle.
    (b) Prove that the function u in C^2(D) satisfying the mean value property at every point D will be harmonic in D.

20. (a) Derive the general solution of one dimensional wave equation.
    (b) Let u(x,t) be the solution of the Cauchy problem
    $u_{tt} - u_{xx} = 0$, $-\infty < x < \infty$, $t > 0$,
    $$u(x,0) = f(x) = \begin{cases} 0 & -\infty < x < -1, \\ x+1 & -1 \leq x \leq 0, \\ 1-x & 0 \leq x \leq 1, \\ 0 & 1 < x < \infty. \end{cases}$$
    $$u_t(x,0) = g(x) = \begin{cases} 0 & -\infty < x < -1, \\ 1 & -1 \leq x \leq 1, \\ 0 & 1 < x < \infty. \end{cases}$$
    i. Evaluate u at the point $(1, \frac{1}{2})$
    ii. Discuss the smoothness of the solution u.

21. (a) Find the eigenvalues and eigenfunctions of the homogeneous integral equations, $y(x) = \lambda \int_0^1 (2x\zeta - 4x^2) y(\zeta) d\zeta$.

(b) Show that $y'' + Ay' + By = 0$, $y(0) = y(1) = 0$,where A and B are constants, leads to the equation

$y(x) = \lambda \int_0^1 k(x,\zeta) y(\zeta) d\zeta$, Where $k(x,\zeta) = \begin{cases} B\zeta(1-x) + Ax - A & \zeta < x, \\ Bx(1-\zeta) + Ax & \zeta > x. \end{cases}$

(2 × 5 = 10 weightage)