D3AMT2402 **(2 Pages)** **Roll. No. ....**

Name ....

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
MATHEMATICS

FMTH3C12 - COMPLEX ANALYSIS

Time: 3 hours **Maximum: 30 weightage**

PART A: Answer *all* questions. Each question carries 1 weightage.

1. Find the image of the lines $x = \alpha$ under the mapping $\cos z$.
2. Prove that the cross ratio remains invariant under Möbius Transformation.
3. Prove that $\sim$ is an equivalence relation on the set of all closed rectifiable curves in a region G.
4. Define $\gamma : [0, 2\pi] \longrightarrow \mathbb{C}$ by $\gamma(t) = e^{it}$. Evaluate $\displaystyle\int_\gamma \frac{1}{z}$ for $z \neq 0$.
5. State Maximum Modulus Theorem.
6. Describe Cauchy's Estimate.
7. State the Residue Theorem.
8. Prove that a function $f : [a, b] \longrightarrow \mathbb{R}$ is convex if and only if the set $A = \{(x, y) : a \leq x \leq b \ \& \ f(x) \leq y\}$ is a convex set.

(8 x 1= 8 weightage)

PART B: Answer *any two* questions from *each unit.* Each question carries 2 weightage.

Unit 1

9. If z_1, z_2, z_3 and z_4 are four distinct points in $\mathbb{C}_\infty$, then prove that their cross ratio is real if and only if all four points lie on a circle.
10. State and prove the chain rule for analytic functions.
11. State and prove Morera's Theorem.

(P.T.O.)

Unit 2

12. State and prove Liouvelle's Theorem.

13. State and prove the Maximum Modulus Theorem.

14. Define the index of a closed rectifiable curve. Prove that its an integer.

Unit 3

15. State and prove the Argument Principle.

16. State and prove the Residue Theorem.

17. Prove that a differentiable function f on $[a, b]$ is convex if and only if f' is increasing.

(6 x 2= 12 weightage)

Part C: Answer *any one* question. Each question carries 5 weightage.

18. Let $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$ have radius of convergence $R > 0$. Then,

    (a) For each $k \geq 1$, prove that the series $\sum_{n=k}^{\infty} n(n-1)\dots(n-k+1)a_n(z-a)^{n-k}$ has the radius of convergence R.

    (b) Prove that f is infinitely differentiable on $B(a; R)$ and that
    $f^{(k)}(z) = \sum_{n=k}^{\infty} n(n-1)\dots(n-k+1)a_n(z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| < R$.

    (c) Prove that $a_n = \frac{1}{n!} f^{(n)}(a)$ for $n \geq 0$.

19. State and prove the first version of Cauchy's Integral Formula.

20. Let G be a connected open set and $f : G \longrightarrow \mathbb{C}$ be an analytic function. Then prove that the following statements are equivalent:

    (a) $f \equiv 0$;

    (b) there is a point $a \in G$ such that $f^{(n)}(a) = 0$ for each $n \geq 0$;

    (c) $\{z \in G : f(z) = 0\}$ has a limit point in G.

21. State and prove Schwarz Lemma.

(2 x 5= 10 weightage)