D3AST2301 (3 Pages) Name: ....

Reg. No...............................

THIRD SEMESTER M.Sc.DEGREE EXAMINATION, NOVEMBER 2024

(Regular/Improvement/Supplementary)

STATISTICS

FMST3C11: STOCHASTIC PROCESSES

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *any four* questions. Each carries *two* weightage.

1. Define Stochastic process. Illustrate it through an example.
2. Briefly describe discrete time branching process.
3. What do you mean by counting process? Give an example to counting process.
4. The number of accidents in a town follows Poisson process with a mean of 2 per day. If the number of people in the i^{th} accident say X_i has the probability distribution

$$P(X_i = k) = \frac{1}{2^k}, \quad k \geq 1,$$

find the mean and the varience of the number of people involved in accidents per week.

5. Describe birth and death process.
6. What is meant by semi-Markov process?
7. Explain basic characteristics of queueing system.

(4 x 2= 8 weightage)

Part B: Answer *any four* questions. Each carries *three* weightage.

8. Define Markov chain. Let $\{X_n\}$ be a Markov chain with state space $\{0, 1, 2\}$ and initial distribution

$$P(X_0 = i) = \frac{1}{3}, \quad i = 0, 1, 2.$$

The t. p .m. is

$$\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

(P.T.O.)

Find the following probabilities:

i) $P[X_3 = 1]$

ii) $P(X_2 = 2, X_1 = 1, X_0 = 2)$

iii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$

9. State and prove Chapman-Kolmogrov equations.

10. If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson process with parameters λ_1 and λ_2 respectively, show that

$$P(N_1(t) = k | N_1(t) + N_2(t) = n) = \binom{n}{k} p^k q^{n-k}, \quad k = 0(1)n$$

where $p = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = 1 - p = \dfrac{\lambda_2}{\lambda_1 + \lambda_2}$.

11. Write short notes on the following:

i) Homogenous Poisson process.

ii) Compound Poisson process.

12. Define renewal process. Illustrate it through a simple example. What is a renewal function?

13. If $M(t)$ is a renewal function with usual notations, prove that $M(.)$ satisfies the equation

$$M(t) = F(t) + \int_0^t M(t - x) dF(x).$$

14. Define Brownian motion process. Explain variation on Brownian motion.

(4 x 3= 12 weightage)

Part C: Answer *any two* questions. Each carries *five* weightage.

15. a) Show that the matrix of transition probabilities together with the initial distribution completely specify a Markov chain

$$\{X_n, n = 0, 1, 2, ...\}$$

b) Classify the states of a Markov chain whose one step t.p.m is given by

$$\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

16. a) Describe the postulates of Poisson process. Using these postulates, if $\{N(t)\}$ is a Poisson process with mean λt, prove that,

$$P(N(t) = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}, \quad k = 0, 1, 2....$$

b) If $\{N(t)\}$ is a Poisson process and $s, t > 0$ with $s < t$, prove that

$$P(N(s) = k | N(t) = n) = \binom{n}{k} (\frac{s}{t})^k (1 - \frac{s}{t})^{n-k}.$$

17. a) State and prove elementary renewal theorem.

b) Write short note on renewal reward process and regenerative process.

18. a) Explain network of queues.

b) Explain multiserver queueing system and derive their Steady State Probabilities.

(2 x 5= 10 weightage)