(2 Pages)

D3AMT2305

Reg. No..............................
Name: ....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3E01 - CODING THEORY

Time: Three Hours Max. Weightage : 30

Part A

Answer **all** questions.
Each question carries 1 weightage.

1. Define a Binary Symmetric Channel. Explain how to convert a channel with reliability $0 < p \leq \frac{1}{2}$ into a channel with $\frac{1}{2} \leq p < 1$.
2. Calculate $\Phi_{.97}(111101, 000010)$.
3. Distinguish CMLD and IMLD.
4. Define perfect codes. Give example.
5. Does there exists a linear code of length $n = 9$ dimension $k = 2$ and distance $d = 5$? Justify.
6. Verify Hamming Bound for the linear code C with generator matrix G $= \begin{pmatrix} 100111 \\ 010101 \\ 001011 \end{pmatrix}$.
7. Find the number of proper linear codes of length $n = 4$.
8. Let $g(x) = 1 + x^2 + x^3$ be the generator polynomial of a linear cyclic code of length 7. Encode $x + x^2 + x^3$ and find the corresponding message.

(8 x 1 = 8 weightage)

Part B

Answer any **two** questions from **each unit**.
Each question carries 2 weightage.

Unit I

9. Let $C = \{000, 111\}$,p=0.9. If $v = 000$ is sent, find the probability that IMLD will correctly concludes this after one transmission.

(P.T.O.)

10. Define basis of a linear code. Find a basis for the linear code $C =< S >$ where $S = \{11101, 10110, 01011, 11010\}$.

11. Find a basis for $C^{\perp}$ for the code $C =< 0101, 1010, 1100 >$.

Unit II

12. Find generating and parity check matrix for a Hamming code of length 7 and decode $w = 1010101$.

13. Show that $\dim RM(r, m) = \sum_{i=0}^{r} {}^{m}\mathrm{C}_i$.

14. Find the generator matrix $\mathrm{G}(2, 3)$ for $RM(2, 3)$

Unit III

15. Let C be a linear cyclic code of length 7 with generator polynomial $g(x) = 1+x+x^3$. Find a generator matrix for C.

16. Show that if C is a cyclic linear code of length n and dimension k with generator $g(x)$ and $1+x^n = g(x)h(x)$, then $\mathrm{C}^{\perp}$ is cyclic code of dimension $n-k$ with generator $x^k h(x^{-1})$.

17. Determine whether $c = 011001011000010$ is a codeword in C_{15}, where $g(x) = 1 + x^4 + x^6 + x^7 + x^8$.

(6 x 2 = 12 weightage)

Part C

Answer any **two** questions. Each carries 5 weightage.

18. a) Define distance of a code. Show that a code of distance d will correct all error patters of weight less than or equal to $\lfloor \frac{(d-1)}{2} \rfloor$. Moreover, there is atleast one error pattern of weight $1 + \lfloor \frac{(d-1)}{2} \rfloor$ which C will not correct.

    b) Let $\mathrm{C} = \{000, 111\}$. Find all error petters that C will correct.

19. Construct an SDA for C with generator matrix $\mathrm{G} = \begin{pmatrix} 100110 \\ 010011 \\ 001111 \end{pmatrix}$ and hence decode $w = 110000$.

20. Decode $w = 001001001001, 11111110000$ which is encoded using C_{23}.

21. Find a parity check matrix for a cyclic Hamming code of length 7 using $\mathrm{GF}(2^3)$ constructed with $1 + x + x^3$ where generator polynomial is $m_3(x)$.
    If $w(x) = x + x^2 + x^4$, find the most likely codeword $c(x)$.

(2 x 5 = 10 weightage)