D3AMT2304 **(2 Pages)** **Name............................**

Reg.No..........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3C14 - PDE AND INTEGRAL EQUATIONS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries *1* weightage.

1. Give example of a hyperbolic type partial differential equation and show that it is of hyperbolic type.
2. Using suitable characteristics, write the general solution of the wave equation.
3. Explain the Domain of Dependence for a Cauchy problem with an example.
4. Write the two dimensional Laplace's equation. Identify its type.
5. What is the separation of variables method for solution of a partial differential equation?
6. Define an integral equation. Give one example of a Fredholm type integral equation.
7. Define separable kernel of an integral equation. Give one example of such a kernel.
8. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real.
.

(8 x 1= 8 weightage)

Part B: Answer *any two* questions from *each unit.* Each carries *2* weightage.
Unit I

9. Find a and b if $u(x,y) = f(ax+by)$ is a solution of the equation $3u_x - 7u_y = 0$.
10. Consider a quasi linear equation $a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u)$
and an initial curve Γ corresponding to the initial conditions
$x(0,s) = x_0(s)$, $y(0,s) = y_0(s)$, $u(0,s) = u_0(s)$.
Explain the meaning that the quasi linear equation and Γ satisfy the transversality condition at a point s on Γ.
11. Find a canonical transformation $q = q(x,y)$, $r = r(x,y)$ and obtain the canonical form of the partial differential equation $u_{xx} + xu_{yy} = 0$, $x > 0$.

Unit II

12. Prove the uniqueness of solution of the Neumann Problem
$u_{tt} - c^2u_{xx} = F(x,t) \quad 0 < x < L, t > 0$
$u_x(0,t) = a(t)$, $u_x(L,t) = b(t)$, $t \geq> 0$
$u(x,0) = f(x), \quad u_t(x,0) = g(x)$, $0 \leq x \leq L$.

(P.T.O.)

13. State and prove the weak maximum principle of a harmonic function.

14. Let u be a function in $C^2(D)$ satisfying the mean value property at every point in D. Then prove that u is harmonic in D.

Unit III

15. Transform the boundary value problem
$\frac{d^2y}{dx^2} + 4y = 0$, $y(0) = 1$, $y(1) = 0$ into an integral equation.

16. Using Green's function, solve the boundary value problem
$\frac{d^2y}{dx^2} = f(x)$, $y(0) = 0 = y(1)$.

17. Explain the method of solving a Fredholm integral equation having a separable kernel.

(6 x 2= 12 weightage)

Part C: Answer *any two* questions. Each carries *5* weightage.

18. Prove that $u(x, y) = x + y^{1/3}$ is the explicit solution to the PDE
$u_x + 3y^{2/3}u_y = 2$ subject to the initial condition $u(x, 1) = 1 + x$.

19. Solve the Cauchy Problem
$u_{tt} - c^2u_{xx} = 0 \quad -\infty < x < \infty,\ t > 0;$
$u(x.0) = f(x) = 1$ if $-2 \leq x \leq 2$ and 0 otherwise $\quad -\infty < x < \infty$,
$u_t(x, 0) = g(x) = 1$ if $-2 \leq x \leq 2$ and 0 otherwise $\quad -\infty < x < \infty$.

20. Write the heat equation. Describe the method of separation of variables for solving heat equation.

21. Solve the integral equation $y(x) = F(x) + \lambda \int_0^1 (1 - 3x\xi)y(\xi)d\xi$ based on the fact that it has a separable kernel.

(2 x 5 = 10 weightage)