THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)
STATISTICS
FMST3C12 - TESTING OF STATISTICAL HYPOTHESES

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define $\alpha-$similar tests. Explain tests with Neyman structure.

2. Distinguish between level of significance and size of a test

3. Define invariant test.

4. Define an unbiased test. Show that a UMP test is unbiased.

5. State Karlin Rubin theorem.

6. What do you mean by p-value?

7. List out the advantages and disadvantages of using non-parametric tests.

(4 x 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define most powerful test. A random sample of size 50 is taken from $N(\theta, 25)$. Find the most powerful test for testing $H_0 : \theta = 10$ vs $H_1 : \theta = 15$.

9. A sample of size 1 is taken from an exponential PDF with parameter θ. To test $H_0 : \theta = 1$ against $H_1 : \theta > 1$, the test to be used is the nonrandomized test

$$\phi(x) = \left\{ \begin{array}{ll} 1, & if\ x > 2; \\ 0, & if\ x \leq 2. \end{array} \right.$$

Find the size of the test. What is the power function?

10. In the first proof of 392 pages of a book, the distribution of printing mistakes were found to be as follows:
No. of mistakes per page: 0   1   2   3   4   5   6
          No. of pages: 275 72 30   7   5   2   1
Fit a Poisson distribution to the above data and test the goodness of fit.

11. Define MLR property. Obtain the UMP test for testing $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$ based on a random sample of size n from $U(0, \theta)$, $\theta > 0$.

12. Obtain the Neyman-Pearson most powerful level α test for $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ for the exponential distribution with mean θ.

(P.T.O.)

13. Define SPRT. Show that SPRT terminates eventually with probability one.

14. Use Wilcoxon's signed-rank test to test $H_0 : M_1 - M_2 = 0$ against $H_1 : M_1 - M_2 \neq 0$ at 0.05 level of significance.
    x_i: 71 108 72 141 61 97 90 127
    y_i: 77 105 71 152 88 117 93 130

(4 x 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. a) Write a short note on families with monotone likelihood ratio. Check whether $U(0, \theta)$ has an MLR.
    b) Define one parameter exponential family. Show that one-parameter exponential family has an MLR.

16. Let $X_1, X_2, ..., X_n$ and $Y_1, Y_2, ..., Y_n$ be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Obtain likelihood ratio test for testing $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$.

17. a) Explain chi-square test for homogeneity.

    b) Define one sample Kolmogorov-Smirnov test statistic. Show that it is distribution free.

18. Define the OC function and ASN function in sequential analysis. Derive their approximate expressions for the SPRT of a simple hypothesis against a simple alternative.

(2 x 5 = 10 weightage)