D3AST2201 (2 Pages) Name.............................
Reg.No..........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

STATISTICS
FMST3C11 - STOCHASTIC PROCESSES

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define a transition probability matrix (t.p.m) of a Markov chain (MC). Show that n-step t.p.m of a Markov chain is n^{th} power of one-step t.p.m
2. Define: (a) Regenerative process and (b) Renewal reward process.
3. "Communication is an equivalence relationship". Establish your claim regarding the truth of this statement.
4. Show that the sum of two independent Poisson processes is again a Poisson process.
5. Define conditional mixed Poisson process and non-homogeneous Poisson process.
6. Describe an Multi server queues.
7. What do you mean by a Gaussian process? Explain its uses in stochastic modelling?

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define stationary distribution of a MC and show that for an ergodic MC the stationary distribution exists uniquely.
9. For the MC with states 0,1,and 2 and t.p.m

$$\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

show that: (a) the MC is irreducuibe.

(b) The states are periodic with period 2.

(c) $\lim_{n\to\infty} {P_{11}}^{2n} = 1.$

10. Define a compound Poisson process. Obtain the probability generating function and hence the mean and variance.

(P.T.O.)

11. (a) Define a birth and death process $\{X(t), t \geq 0\}$ with birth rate $\{\lambda_n\}$ and death rate $\{\mu_n\}$.

(b) If $P_n(t) = P(X(t) = n/X(0) = i)$, derive the differential equation satisfied by $P_n(t)$ for the birth and death process in (a).

12. Describe a renewal reward process. Show that, with probability one,

$\lim_{t\to\infty} \frac{R(t)}{t} = \frac{E(R)}{E(X)}$ (under usual notations with $E(R) < \infty, E(X) < \infty$.).

13. (a) Distinguish between a Markov process and a semi-Markov process.

(b) State and prove the elementary renewal theorem.

14. Derive the steady state solution for the number of customers in the system in a M/M/1 queue. Obtain the expected number of customers in the queue.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. (a) Let E and F are two Poisson processes with mean ‘at’ and ‘bt’ respectively. Show that the number N of occurrences of E between two successive occurrences of F has geometric distribution.

(b) If $\{N(t), t \geq 0\}$ is a Poisson process, show that the auto correlation between N(t) and N(t+s) is $\left(\frac{t}{t+s}\right)^{1/2}$.

16. Suppose that the interarrival distribution of a renewal process $\{N(t), t \geq 0\}$ is Poisson distributed with mean λ. Calculate $P(N(t) = n)$. Find the distribution of the time of the n^{th} renewal.

17. Describe Galton Watson branching process $\{X_n\}$, with offspring distribution having probability generating function (pgf)$\emptyset(s)$, show that in the usual notations

$\emptyset_{n+1}(s) = \emptyset_n(\emptyset(s))$. Also find $E(X_n)$ and $Var(X_n)$.

18. Describe the M/G/1 and G/M/1 queueing systems. Discuss the steady state solution in M/G/1 model.

(2 × 5 = 10 weightage)