D3AMT2205 **(2 Pages)** **Name..........................**

Reg.No.........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3E01 - CODING THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define Maximum likelihood decoding[MLD] and its types.
2. Check whether $S = \{1101, 0111, 1100, 0011\}$ is linearly independent or not?
3. Find the total number of bases of a linear code K^4 with dimension 4.
4. Prove that a Hamming code has distance 3.
5. Does there exist a $(15, 7, 5)$ linear code?
6. Show that the cyclic shift π is a linear transformation.
7. Find a basis for the smallest linear cyclic code of length 4 containing $v = 0101$.
8. Prove that Golay code $(23, 12, 7)$ is perfect.

(8 x 1 = 8 weightage)

Part B: Answer *any two* questions *from each unit.* Each carries *2* weightage.

UNIT I

9. Define the reliability of a BSC. What can be said about a channel when
   (a) its reliability is 0.
   (b) its reliability is $\frac{1}{2}$.
10. Show that a code C of distance d will correct all error patterns of weight less than or equal to $\lfloor \frac{d-1}{2} \rfloor$ and there is atleast one error pattern of weight $1 + \lfloor \frac{d-1}{2} \rfloor$ which C will not correct.
11. Given is a parity-check matrix for a linear code C. $H = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Find generator matrices for C and $C^{\perp}$.

(P.T.O.)

UNIT II

12. Prove the following

    (a) A Hamming code has dimension $2^r - 1 - r$ and contains 2^{2^r-1-r} codewords.
    (b) Hamming codes are perfect single-error correcting codes.

13. For the Hamming code of $n = 7$ decode the message $w = 1101001$.

14. Find generating and parity check matrices for an extended Hamming code of length 8.

UNIT III

15. Prove that every cyclic code contains a unique idempotent polynomial which generates the code.

16. Let C be a cyclic code of length n and let $g(x)$ be the generator polynomial. If $n-k$ =degree $(g(x))$ then prove the following:

    (a) C has dimension k.
    (b) The code words corresponding to $g(x), xg(x), \ldots, x^{k-1}g(x)$ form a basis for C.

17. Let $\alpha \neq 0$ be an element of $GF(2^r)$. Let $m_\alpha(x)$ be the minimal polynomial of α. Then prove the following.

    (a) $m_\alpha(x)$ is irreducible over K
    (b) If $f(x)$ is any polynomial over K such that $f(\alpha) = 0$, then $m_\alpha(x)$ is a factor of $f(x)$
    (c) the minimal polynomial is unique
    (d) the minimal polynomial $m_\alpha(x)$ is a factor of $1 + x^{2^r-1}$.

(6 x 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

18. (a) If $C = \{01000, 01001, 00011, 11001\}$ and a word $w = 10110$ is received, which code word was most likely sent?
    (b) Let $|M| = 3$ and $n = 3$. For each word w in K^3 that could be received, find the word v in the code $C = \{000, 001, 110\}$ which IMLD will conclude was sent.

19. Prove that the r^{th} order Reed- Muller code $RM(r, m)$ has following properties:

    (a) distance $d = 2^{m-r}$
    (b) dimension $k = \sum_{i=0}^{r} \binom{m}{i}$
    (c) $RM(r-1, m)$ is contained in $RM(r, m), r > 0$
    (d) dual code $RM(m-1-r, m), r < m$.

20. Find all primitive elements in $GF(2^4)$.

21. Decode the received message $w = 110111101011000$ using the BCH code C_{15}.

(2 x 5 = 10 weightage)