D3AMT2204 **(2 Pages)** **Name**............................

Reg.No.........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3C14 - PDE AND INTEGRAL EQUATIONS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries *1* weightage.

1. Give example of a parabolic type partial differential equation and show that it is of parabolic type.
2. Obtain the canonical form of one dimensional wave equation.
3. Explain the Region of Influence for a Cauchy problem with an example.
4. State the heat equation and explain the homogeneous boundary conditions of it.
5. State the maximum principle of heat equation.
6. Define an integral equation.
   Give one example of a Volterra type integral equation.
7. Exlain the use of Green's function to solve a differential equation with suitable boundary conditions.
8. Describe the iterative method to solve an integral equation of the second kind.

(8 x 1 = 8 weightage)

Part B: Answer *any two* questions *from each unit.* Each carries *2* weightage.

Unit II

9. Find a and b if $u(x, y) = f(ax + by)$ is a solution of the equation $u_x + 3u_y = 0$.
10. Explain the characteristic equations associated with the linear partial differential equation $a(x, y)u_x + b(x, y)u_y = c_0(x, y)u + c_1(x, y)$ where a, b, c_0 & c_1 are given functions of x and y.
11. Prove that the equation $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$ is parabolic and find its canonical form. Hence write the solution of the equation.

Unit II

12. Explain the energy method associated with the Neumann Problem of a vibrating string.

(P.T.O)

13. Prove that a necessary condition for the existence of a solution to the Neumann problem is $\int\limits_{\partial D} g(x(s), y(s))ds = \int\limits_{D} F(x, y)dx\,dy$
where $(x(s), y(s))$ is a parametrisation of ∂D.

14. State and prove the mean value principle of a harmonic function.

Unit III

15. Transform the initial value problem $\frac{d^2y}{dx^2} + y = x$, $y(0) = 1$, $y'(0) = 0$ into an integral equation.

16. State and prove the Abel's formula associated to the differential equation

$$p\frac{d^2y}{dx^2} + \frac{dp}{dx}\frac{dy}{dx} + qy = 0.$$

17. Define the terms 'characteristic number' and 'characteristic function' of a homogeneous Fredholm integral equation $y(x) = \lambda \int\limits_0^1 K(x, \xi)y(\xi)d\xi$.
If the kernel is symmetric, what can be concluded about the characteristic numbers of the equation ? Prove your conclusion.

(6 x 2 = 12 weightage)

Part C: Answer *any two* questions. Each carries *5* weightage.

18. Obtain the De Alembert's solution of the one dimensional wave equation
$u_{tt} - c^2u_{xx} = 0 \quad -\infty < x < \infty,\ t > 0;$
$u(x.0) = f(x), \quad u_t(x, 0) = g(x) \quad -\infty < x < \infty.$

19. Write the wave equation. Describe the method of separation of variables for solving wave equation.

20. Solve the equation $u_x + u_y + u = 1$, subject to the initial condition $u = \sin x$, on $y = x + x^2$, $x > 0$.

21. Solve the Fredholm integral equation $y(x) = 1+\lambda \int\limits_0^1 (1-3x\xi)y(\xi)d\xi$ by the method of successive approximations.

(2 x 5 = 10 weightage)