D3AMT2203 (2 Pages) Name.............................

Reg.No..........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH3C13- FUNCTIONAL ANALYSIS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. Show that two cosets $[x]$ and $[y]$ of a linear space E either coincide or they are disjoint.
2. Prove that every bounded operator of finite rank is a compact operator.
3. Prove that C_0 is a closed subspace of l_∞.
4. Let E be a Banach Space and let E_1 be a subspace of E. Then prove that E_1 is a Banach Space if and only if E_1 is closed.
5. Show that inner product is a continuous function with respect to both variables.
6. If L is a closed subspace of H, then prove that $(L^T)^T = L$
7. If $f \in E^H/_{\{0\}}$, show that $codim \ker f = 1$.
8. State and prove Cauchy Schwartz inequality.

(8 × 1 = 8 weightage)

Part B: Answer *any two* questions *from each unit*. Each carries 2 weightage.

Unit 1

9. Prove that the dimension of $E/_{E_1}$ is n if and only if there exists $x_1, x_2, \dots, x_n$ *linearly independent* vectors relative to E_1 such that for every $x \in E$ *there exist* a unique set of numbers $a_1, a_2, \dots, a_n$ and a unique vector $y \in E_1$ such that $x = \sum_1^n x_i a_i$ + y.
10. Show that the kernel for a seminorm p is a subspace of a linear space .on which it is defined. Also show that $p(x+y)$ is independent of y , where y is an element of the subspace.
11. Show that the space l_p for $1 \leq p < \infty$ is a Banach Space.

(P.T.O.)

Unit 2

12. State and prove Bessel's Inequality.

13. Prove that the Hilbert space H is separable if and only if there exists a complete orthonormal system for H.

14. Give an example of a non separable Hilbert space. Justify your claim.

Unit 3

15. The l_p spaces are reflexive. Justify the statement.

16. Let X and Y be normed spaces and $K(X,Y)$ be the set of all compact operators from X to Y. Prove that $K(X,Y)$ is a closed linear subspace of $L(X,Y)$.

17. Let $K(t,\tau)$ be a continuous function of two variables on $[0,1] \times [0,1]$. Prove that the operator $Kx = \int_0^1 K(t,\tau)x(\tau)\,dx$ is a compact operator on $C[0,1]$.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

18. State and prove Holder's inequality and Minkowski's inequality.

19. (a) State and prove Parsevals identity.

    (b) State and prove Reisz Representation theorem.

20. (a) Prove that the dual of C_0 is linearly isometric to l_1 .

    (b) Show that every finite dimensional normed space is a reflexive space.

21. Let X be a normed space and let Y be a Banach space. Then prove that $L(X,Y)$ is a Banach space.

(2 × 5 = 10 weightage)